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Preface

Welcome to *Elementary Algebra*, an OpenStax resource. This textbook was written to increase student access to high-quality learning materials, maintaining highest standards of academic rigor at little to no cost.

About OpenStax

OpenStax is a nonprofit based at Rice University, and it's our mission to improve student access to education. Our first openly licensed college textbook was published in 2012, and our library has since scaled to over 25 books for college and AP courses used by hundreds of thousands of students. Our adaptive learning technology, designed to improve learning outcomes through personalized educational paths, is being piloted in college courses throughout the country. Through our partnerships with philanthropic foundations and our alliance with other educational resource organizations, OpenStax is breaking down the most common barriers to learning and empowering students and instructors to succeed.

About OpenStax Resources

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Format

You can access this textbook for free in web view or PDF through openstax.org, and for a low cost in print.

About *Elementary Algebra*

Elementary Algebra is designed to meet the scope and sequence requirements of a one-semester elementary algebra course. The book’s organization makes it easy to adapt to a variety of course syllabi. The text expands on the fundamental concepts of algebra while addressing the needs of students with diverse backgrounds and learning styles. Each topic builds upon previously developed material to demonstrate the cohesiveness and structure of mathematics.

Coverage and Scope

Elementary Algebra follows a nontraditional approach in its presentation of content. Building on the content in *Prealgebra*, the material is presented as a sequence of small steps so that students gain confidence in their ability to succeed in the course. The order of topics was carefully planned to emphasize the logical progression through the course and to facilitate a thorough understanding of each concept. As new ideas are presented, they are explicitly related to previous topics.

Chapter 1: Foundations
Chapter 1 reviews arithmetic operations with whole numbers, integers, fractions, and decimals, to give the student a solid base that will support their study of algebra.

Chapter 2: Solving Linear Equations and Inequalities
In Chapter 2, students learn to verify a solution of an equation, solve equations using the Subtraction and Addition Properties of Equality, solve equations using the Multiplication and Division Properties of Equality, solve equations with variables and constants on both sides, use a general strategy to solve linear equations, solve equations with fractions or decimals, solve a formula for a specific variable, and solve linear inequalities.

Chapter 3: Math Models
Once students have learned the skills needed to solve equations, they apply these skills in Chapter 3 to solve word and number problems.

Chapter 4: Graphs
Chapter 4 covers the rectangular coordinate system, which is the basis for most consumer graphs. Students learn to plot points on a rectangular coordinate system, graph linear equations in two variables, graph with intercepts, understand slope of a line, use the slope-intercept form of an equation of a line, find the equation of a line, and create graphs of linear inequalities.

Chapter 5: Systems of Linear Equations
Chapter 5 covers solving systems of equations by graphing, substitution, and elimination; solving applications with systems of equations, solving mixture applications with systems of equations, and graphing systems of linear inequalities.

Chapter 6: Polynomials
In Chapter 6, students learn how to add and subtract polynomials, use

multiplication properties of exponents, multiply polynomials, use special products, divide monomials and polynomials, and understand integer exponents and scientific notation.

Chapter 7: Factoring

In Chapter 7, students explore the process of factoring expressions and see how factoring is used to solve certain types of equations.

Chapter 8: Rational Expressions and Equations

In Chapter 8, students work with rational expressions, solve rational equations, and use them to solve problems in a variety of applications.

Chapter 9: Roots and Radical

In Chapter 9, students are introduced to and learn to apply the properties of square roots, and extend these concepts to higher order roots and rational exponents.

Chapter 10: Quadratic Equations

In Chapter 10, students study the properties of quadratic equations, solve and graph them. They also learn how to apply them as models of various situations.

All chapters are broken down into multiple sections, the titles of which can be viewed in the **Table of Contents**.

Key Features and Boxes

Examples Each learning objective is supported by one or more worked examples that demonstrate the problem-solving approaches that students must master. Typically, we include multiple Examples for each learning objective to model different approaches to the same type of problem, or to introduce similar problems of increasing complexity.

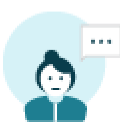
All Examples follow a simple two- or three-part format. First, we pose a problem or question. Next, we demonstrate the solution, spelling out the steps along the way. Finally (for select Examples), we show students how to check the solution. Most Examples are written in a two-column format, with explanation on the left and math on the right to mimic the way that instructors “talk through” examples as they write on the board in class.

Be Prepared! Each section, beginning with Section 2.1, starts with a few "Be Prepared!" exercises so that students can determine if they have mastered the prerequisite skills for the section. Reference is made to specific Examples from previous sections so students who need further review can easily find explanations. Answers to these exercises can be found in the supplemental resources that accompany this title.

Try It

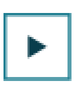
The Try It feature includes a pair of exercises that immediately follow an Example, providing the student with an immediate opportunity to solve a similar problem. In the Web View version of the text, students can click an Answer link directly below the question to check their understanding. In the PDF, answers to the Try It exercises are located in the Answer Key.

How To



How To feature typically follows the Try It exercises and outlines the series of steps for how to solve the problem in the preceding Example.

Media



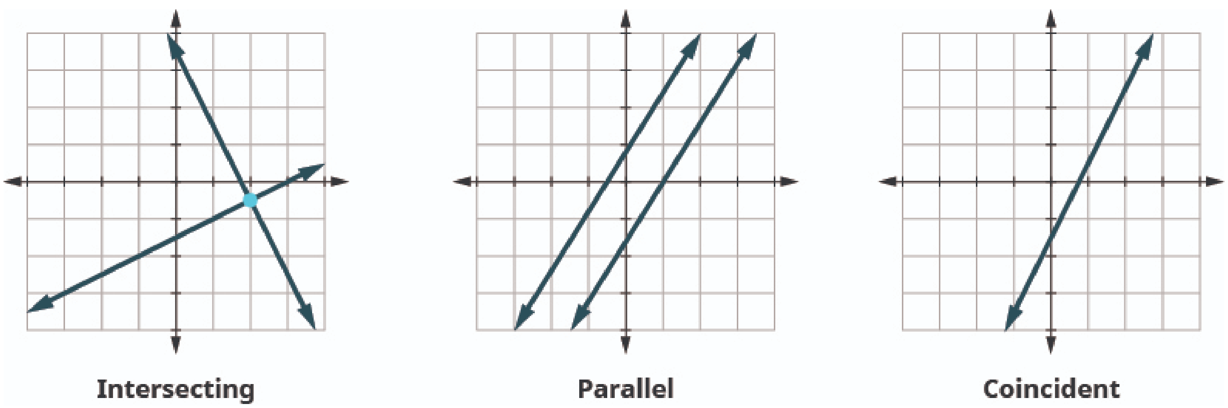
The Media icon appears at the conclusion of each section, just prior to the Self Check. This icon marks a list of links to online video tutorials that reinforce the concepts and skills introduced in the section.

Disclaimer: While we have selected tutorials that closely align to our learning objectives, we did not produce these tutorials, nor were they specifically produced or tailored to accompany *Elementary Algebra*.

Self Check The Self Check includes the learning objectives for the section so that students can self-assess their mastery and make concrete plans to improve.

Art Program

Elementary Algebra contains many figures and illustrations. Art throughout the text adheres to a clear, understated style, drawing the eye to the most important information in each figure while minimizing visual distractions.



Section Exercises and Chapter Review

Section Exercises Each section of every chapter concludes with a well-rounded set of exercises that can be assigned as homework or used selectively for guided practice. Exercise sets are named *Practice Makes Perfect* to encourage completion of homework assignments.

- Exercises correlate to the learning objectives. This facilitates assignment of personalized study plans based on individual student needs.
- Exercises are carefully sequenced to promote building of skills.
- Values for constants and coefficients were chosen to practice and reinforce arithmetic facts.
- Even and odd-numbered exercises are paired.

Exercises parallel and extend the text examples and use the same instructions as the examples to help students easily recognize the connection.
Applications are drawn from many everyday experiences, as well as those traditionally found in college math texts.
Everyday Math highlights practical situations using the concepts from that particular section
Writing Exercises are included in every exercise set to encourage conceptual understanding, critical thinking, and literacy.

Chapter Review Each chapter concludes with a review of the most important takeaways, as well as additional practice problems that students can use to prepare for exams.

Key Terms provide a formal definition for each bold-faced term in the chapter.
Key Concepts summarize the most important ideas introduced in each section, linking back to the relevant Example(s) in case students need to review.
Chapter Review Exercises include practice problems that recall the most important concepts from each section.
Practice Test includes additional problems assessing the most important learning objectives from the chapter.
Answer Key includes the answers to all Try It exercises and every other exercise from the Section Exercises, Chapter Review Exercises, and Practice Test.

Additional Resources

Student and Instructor Resources

We’ve compiled additional resources for both students and instructors, including Getting Started Guides, manipulative mathematics worksheets, and an answer key to Be Prepared Exercises. Instructor resources require a verified instructor account, which can be requested on your openstax.org log-in. Take advantage of these resources to supplement your OpenStax book.

Partner Resources

OpenStax Partners are our allies in the mission to make high-quality learning materials affordable and accessible to students and instructors everywhere. Their tools integrate seamlessly with our OpenStax titles at a low cost. To access the partner resources for your text, visit your book page on openstax.org.

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# Introduction
class="introduction"

In order to be structurally sound, the foundation of a building must be carefully constructed.

Just like a building needs a firm foundation to support it, your study of algebra needs to have a firm foundation. To ensure this, we begin this book with a review of arithmetic operations with whole numbers, integers, fractions, and decimals, so that you have a solid base that will support your study of algebra.

Introduction to Whole Numbers
By the end of this section, you will be able to:

- Use place value with whole numbers
- Identify multiples and and apply divisibility tests
- Find prime factorizations and least common multiples

Note:
A more thorough introduction to the topics covered in this section can be found in *Prealgebra* in the chapters **Whole Numbers** and **The Language of Algebra**.

As we begin our study of elementary algebra, we need to refresh some of our skills and vocabulary. This chapter will focus on whole numbers, integers, fractions, decimals, and real numbers. We will also begin our use of algebraic notation and vocabulary.

Use Place Value with Whole Numbers

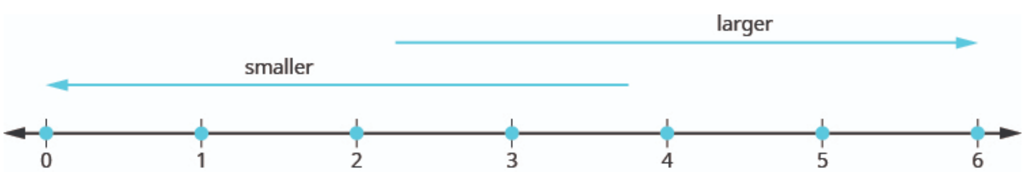
The most basic numbers used in algebra are the numbers we use to count objects in our world: 1, 2, 3, 4, and so on. These are called the **counting numbers**. Counting numbers are also called *natural numbers*. If we add zero to the counting numbers, we get the set of **whole numbers**.

Counting Numbers: 1, 2, 3, …

Whole Numbers: 0, 1, 2, 3, …

The notation "…" is called ellipsis and means "and so on," or that the pattern continues endlessly.

We can visualize counting numbers and whole numbers on a **number line** (see [link]).



The numbers on the number line get larger as they go from left to right, and smaller as they go from right to left. While this number line shows only the whole numbers 0 through 6, the numbers keep going without end.

Note: Doing the Manipulative Mathematics activity "Number Line-Part 1" will help you develop a better understanding of the counting numbers and the whole numbers.

Our number system is called a place value system, because the value of a digit depends on its position in a number. [link] shows the place values. The place values are separated into groups of three, which are called

periods. The periods are *ones, thousands, millions, billions, trillions,* and so on. In a written number, commas separate the periods.

Place Value														
Trillions			Billions			Millions			Thousands			Ones		
Hundred trillions	Ten trillions	Trillions	Hundred billions	Ten billions	Billions	Hundred millions	Ten millions	Millions	Hundred thousands	Ten thousands	Thousands	Hundreds	Tens	Ones
								5	2	7	8	1	9	4

The number 5,278,194 is shown in the chart. The digit 5 is in the millions place. The digit 2 is in the hundred-thousands place. The digit 7 is in the ten-thousands place. The digit 8 is in the thousands place. The digit 1 is in the hundreds place. The digit 9 is in the tens place. The digit 4 is in the ones place.

Example:
Exercise:

Problem: In the number 63,407,218, find the place value of each digit:

ⓐ 7
ⓑ 0
ⓒ 1
ⓓ 6
ⓔ 3

Solution:
Solution

Place the number in the place value chart:

Trillions			Billions			Millions			Thousands			Ones		
Hundred trillions	Ten trillions	Trillions	Hundred billions	Ten billions	Billions	Hundred millions	Ten millions	Millions	Hundred thousands	Ten thousands	Thousands	Hundreds	Tens	Ones
							6	3	4	0	7	2	1	8

ⓐ The 7 is in the thousands place.
ⓑ The 0 is in the ten thousands place.
ⓒ The 1 is in the tens place.
ⓓ The 6 is in the ten-millions place.
ⓔ The 3 is in the millions place.

Note:
Exercise:

Problem: For the number 27,493,615, find the place value of each digit:

ⓐ 2 ⓑ 1 ⓒ 4 ⓓ 7 ⓔ 5

Solution:

ⓐ ten millions ⓑ tens ⓒ hundred thousands ⓓ millions ⓔ ones

Note:
Exercise:

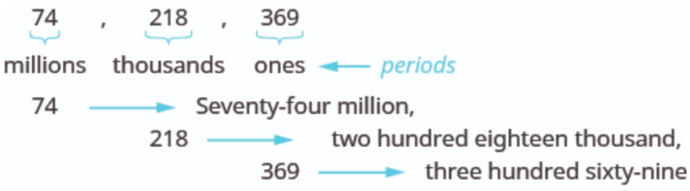
Problem: For the number 519,711,641,328, find the place value of each digit:

ⓐ 9 ⓑ 4 ⓒ 2 ⓓ 6 ⓔ 7

Solution:

ⓐ billions ⓑ ten thousands ⓒ tens ⓓ hundred thousands ⓔ hundred millions

When you write a check, you write out the number in words as well as in digits. To write a number in words, write the number in each period, followed by the name of the period, without the *s* at the end. Start at the left, where the periods have the largest value. The ones period is not named. The commas separate the periods, so wherever there is a comma in the number, put a comma between the words (see [link]). The number 74,218,369 is written as seventy-four million, two hundred eighteen thousand, three hundred sixty-nine.



Note:
Name a Whole Number in Words.

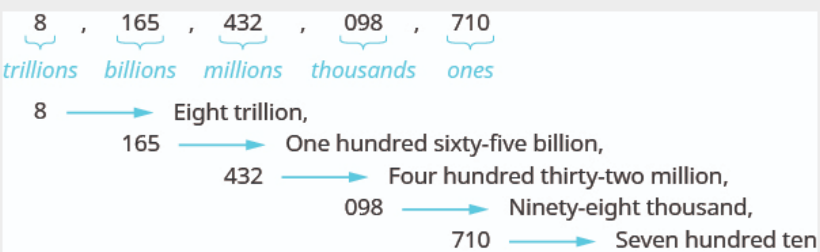
Start at the left and name the number in each period, followed by the period name.
Put commas in the number to separate the periods.
Do not name the ones period.

Example:
Exercise:

Problem: Name the number 8,165,432,098,710 using words.

Solution:
Solution

Name the number in each period, followed by the period name.



Put the commas in to separate the periods.

So, $8,165,432,098,710$ is named as eight trillion, one hundred sixty-five billion, four hundred thirty-two million, ninety-eight thousand, seven hundred ten.

Note:
Exercise:

Problem: Name the number $9,258,137,904,061$ using words.

Solution:

nine trillion, two hundred fifty-eight billion, one hundred thirty-seven million, nine hundred four thousand, sixty-one

Note:
Exercise:

Problem: Name the number $17,864,325,619,004$ using words.

Solution:

seventeen trillion, eight hundred sixty-four billion, three hundred twenty-five million, six hundred nineteen thousand four

We are now going to reverse the process by writing the digits from the name of the number. To write the number in digits, we first look for the clue words that indicate the periods. It is helpful to draw three blanks for the needed periods and then fill in the blanks with the numbers, separating the periods with commas.

Note:
Write a Whole Number Using Digits.

Identify the words that indicate periods. (Remember, the ones period is never named.)
Draw three blanks to indicate the number of places needed in each period. Separate the periods by commas.
Name the number in each period and place the digits in the correct place value position.

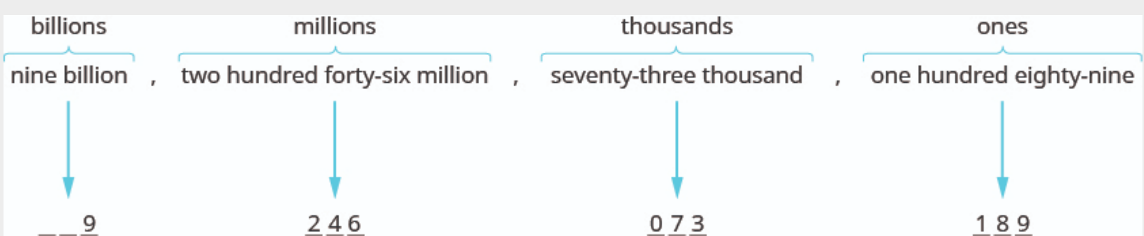
Example:
Exercise:

Problem:

Write *nine billion, two hundred forty-six million, seventy-three thousand, one hundred eighty-nine* as a whole number using digits.

Solution:
Solution

Identify the words that indicate periods.
Except for the first period, all other periods must have three places. Draw three blanks to indicate the number of places needed in each period. Separate the periods by commas.
Then write the digits in each period.



The number is 9,246,073,189.

Note:
Exercise:

Problem:

Write the number two billion, four hundred sixty-six million, seven hundred fourteen thousand, fifty-one as a whole number using digits.

Solution:

2,466,714,051

Note:
Exercise:

Problem:

Write the number eleven billion, nine hundred twenty-one million, eight hundred thirty thousand, one hundred six as a whole number using digits.

Solution:

11,921,830,106

In 2013, the U.S. Census Bureau estimated the population of the state of New York as 19,651,127. We could say the population of New York was approximately 20 million. In many cases, you don’t need the exact value; an approximate number is good enough.

The process of approximating a number is called rounding. Numbers are rounded to a specific place value, depending on how much accuracy is needed. Saying that the population of New York is approximately 20 million means that we rounded to the millions place.

Example:
How to Round Whole Numbers
Exercise:

Problem: Round 23,658 to the nearest hundred.

Solution:
Solution

Step 1. Locate the given place value with an arrow. All digits to the left do not change.	Locate the hundreds place in 23,658.	hundredths place $\downarrow$ 23,658
Step 2. Underline the digit to the right of the given place value.	Underline the 5, which is to the right of the hundreds place.	hundredths place $\downarrow$ $23{,}6\underline{5}8$

Step 3. Is this digit greater than or equal to 5? Yes–add 1 to the digit in the given place value. No–do <u>not</u> change the digit in the given place value.	Add 1 to the 6 in the hundreds place, since 5 is greater than or equal to 5.	23,658 add 1
Step 4. Replace all digits to the right of the given place value with zeros.	Replace all digits to the right of the hundreds place with zeros.	23,700 add 1 replace with 0s So, 23,700 is rounded to the nearest hundred.

Note:
Exercise:

Problem: Round to the nearest hundred: 17,852.

Solution:

17,900

Note:
Exercise:

Problem: Round to the nearest hundred: 468,751.

Solution:

468,800

Note:
Round Whole Numbers.

Locate the given place value and mark it with an arrow. All digits to the left of the arrow do not change.
Underline the digit to the right of the given place value.
Is this digit greater than or equal to 5?

- Yes–add 1 to the digit in the given place value.
- No–do <u>not</u> change the digit in the given place value.

Replace all digits to the right of the given place value with zeros.

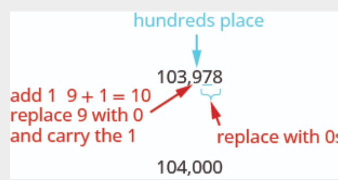
Example:
Exercise:

Problem: Round 103,978 to the nearest:

ⓐ hundred
ⓑ thousand
ⓒ ten thousand

Solution:
Solution

ⓐ

Locate the hundreds place in 103,978.	
Underline the digit to the right of the hundreds place.	
Since 7 is greater than or equal to 5, add 1 to the 9. Replace all digits to the right of the hundreds place with zeros.	
	So, 104,000 is 103,978 rounded to the nearest hundred.

ⓑ

Locate the thousands place and underline the digit to the right of the thousands place.	

Since 9 is greater than or equal to 5, add 1 to the 3. Replace all digits to the right of the hundreds place with zeros.	
	So, 104,000 is 103,978 rounded to the nearest thousand.

ⓒ

Locate the ten thousands place and underline the digit to the right of the ten thousands place.	
Since 3 is less than 5, we leave the 0 as is, and then replace the digits to the right with zeros.	100,000
	So, 100,000 is 103,978 rounded to the nearest ten thousand.

Note:
Exercise:

Problem: Round 206,981 to the nearest: ⓐ hundred ⓑ thousand ⓒ ten thousand.

Solution:

ⓐ 207,000 ⓑ 207,000 ⓒ 210,000

Note:
Exercise:

Problem: Round 784,951 to the nearest: ⓐ hundred ⓑ thousand ⓒ ten thousand.

Solution:

ⓐ 785,000 ⓑ 785,000 ⓒ 780,000

Identify Multiples and Apply Divisibility Tests

The numbers 2, 4, 6, 8, 10, and 12 are called **multiples** of 2. A multiple of 2 can be written as the product of a counting number and 2.

2,	4,	6,	8,	10,	12, …
$2 \cdot 1$,	$2 \cdot 2$,	$2 \cdot 3$,	$2 \cdot 4$,	$2 \cdot 5$,	$2 \cdot 6$

Similarly, a multiple of 3 would be the product of a counting number and 3.

3,	6,	9,	12,	15,	18, …
$3 \cdot 1$,	$3 \cdot 2$,	$3 \cdot 3$,	$3 \cdot 4$,	$3 \cdot 5$,	$3 \cdot 6$

We could find the multiples of any number by continuing this process.

Note: Doing the Manipulative Mathematics activity "Multiples" will help you develop a better understanding of multiples.

[link] shows the multiples of 2 through 9 for the first 12 counting numbers.

Counting Number	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
Multiples of 2	2	4	6	8	10	12	14	16	18	20	22	24
Multiples of 3	3	6	9	12	15	18	21	24	27	30	33	36
Multiples of 4	4	8	12	16	20	24	28	32	36	40	44	48
Multiples of 5	5	10	15	20	25	30	35	40	45	50	55	60
Multiples of 6	6	12	18	24	30	36	42	48	54	60	66	72
Multiples of 7	7	14	21	28	35	42	49	56	63	70	77	84
Multiples of 8	8	16	24	32	40	48	56	64	72	80	88	96

Counting Number	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
Multiples of 9	9	18	27	36	45	54	63	72	81	90	99	108
Multiples of 10	10	20	30	40	50	60	70	80	90	100	110	120

Note:
Multiple of a Number
A number is a **multiple** of *n* if it is the product of a counting number and *n*.

Another way to say that 15 is a multiple of 3 is to say that 15 is **divisible** by 3. That means that when we divide 3 into 15, we get a counting number. In fact, $15 \div 3$ is 5, so 15 is $5 \cdot 3$.

Note:
Divisible by a Number
If a number *m* is a multiple of *n*, then *m* is **divisible** by *n*.

Look at the multiples of 5 in [link]. They all end in 5 or 0. Numbers with last digit of 5 or 0 are divisible by 5. Looking for other patterns in [link] that shows multiples of the numbers 2 through 9, we can discover the following divisibility tests:

Note:
Divisibility Tests
A number is divisible by:

- 2 if the last digit is 0, 2, 4, 6, or 8.
- 3 if the sum of the digits is divisible by 3.
- 5 if the last digit is 5 or 0.
- 6 if it is divisible by both 2 and 3.
- 10 if it ends with 0.

Example:
Exercise:

Problem: Is 5,625 divisible by 2? By 3? By 5? By 6? By 10?

Solution:
Solution

Is 5,625 divisible by 2?	
Does it end in 0, 2, 4, 6, or 8?	No.
	5,625 is not divisible by 2.
Is 5,625 divisible by 3?	
What is the sum of the digits?	$5+6+2+5=18$
Is the sum divisible by 3?	Yes. 5,625 is divisible by 3.
Is 5,625 divisible by 5 or 10?	
What is the last digit? It is 5.	5,625 is divisible by 5 but not by 10.
Is 5,625 divisible by 6?	
Is it divisible by both 2 and 3?	No, 5,625 is not divisible by 2, so 5,625 is not divisible by 6.

Note:
Exercise:

Problem: Determine whether 4,962 is divisible by 2, by 3, by 5, by 6, and by 10.

Solution:

by 2, 3, and 6

Note:
Exercise:

Problem: Determine whether 3,765 is divisible by 2, by 3, by 5, by 6, and by 10.

Solution:

by 3 and 5

Find Prime Factorizations and Least Common Multiples

In mathematics, there are often several ways to talk about the same ideas. So far, we've seen that if *m* is a multiple of *n*, we can say that *m* is divisible by *n*. For example, since 72 is a multiple of 8, we say 72 is divisible by 8. Since 72 is a multiple of 9, we say 72 is divisible by 9. We can express this still another way.

Since $8\cdot 9=72$, we say that 8 and 9 are **factors** of 72. When we write $72=8\cdot 9$, we say we have factored 72.

$$\underbrace{8\cdot 9}_{\text{factors}} = \underbrace{72}_{\text{product}}$$

Other ways to factor 72 are $1\cdot 72, 2\cdot 36, 3\cdot 24, 4\cdot 18,$ and $6\cdot 12$. Seventy-two has many factors: 1, 2, 3, 4, 6, 8, 9, 12, 18, 36, and 72.

Note:
Factors
If $a \cdot b = m$, then a and b are **factors** of m.

Some numbers, like 72, have many factors. Other numbers have only two factors.

Note: Doing the Manipulative Mathematics activity “Model Multiplication and Factoring” will help you develop a better understanding of multiplication and factoring.

Note:
Prime Number and Composite Number
A **prime number** is a counting number greater than 1, whose only factors are 1 and itself.
A **composite number** is a counting number that is not prime. A composite number has factors other than 1 and itself.

Note: Doing the Manipulative Mathematics activity “Prime Numbers” will help you develop a better understanding of prime numbers.

The counting numbers from 2 to 19 are listed in [link], with their factors. Make sure to agree with the “prime” or “composite” label for each!

Number	Factors	Prime or Composite?
2	1,2	Prime
3	1,3	Prime
4	1,2,4	Composite
5	1,5	Prime
6	1,2,3,6	Composite
7	1,7	Prime
8	1,2,4,8	Composite
9	1,3,9	Composite
10	1,2,5,10	Composite

Number	Factors	Prime or Composite?
11	1,11	Prime
12	1,2,3,4,6,12	Composite
13	1,13	Prime
14	1,2,7,14	Composite
15	1,3,5,15	Composite
16	1,2,4,8,16	Composite
17	1,17	Prime
18	1,2,3,6,9,18	Composite
19	1,19	Prime

The **prime numbers** less than 20 are 2, 3, 5, 7, 11, 13, 17, and 19. Notice that the only even prime number is 2.

A composite number can be written as a unique product of primes. This is called the **prime factorization** of the number. Finding the prime factorization of a composite number will be useful later in this course.

Note:
Prime Factorization
The **prime factorization** of a number is the product of prime numbers that equals the number.

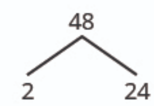
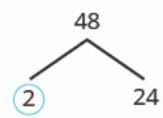
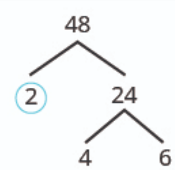
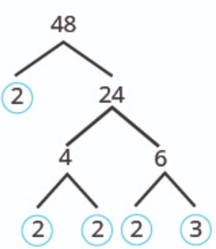
To find the prime factorization of a composite number, find any two factors of the number and use them to create two branches. If a factor is prime, that branch is complete. Circle that prime!

If the factor is not prime, find two factors of the number and continue the process. Once all the branches have circled primes at the end, the factorization is complete. The composite number can now be written as a product of prime numbers.

Example:
How to Find the Prime Factorization of a Composite Number
Exercise:

Problem: Factor 48.

Solution:
Solution

Step 1. Find two factors whose product is the given number. Use these numbers to create two branches.	$48 = 2 \cdot 24$	
Step 2. If a factor is prime, that branch is complete. Circle the prime.	2 is prime. Circle the prime.	
Step 3. If a factor is not prime, write it as the product of two factors and continue the process.	24 is not prime. Break it into 2 more factors.	
	4 and 6 are not prime. Break them each into two factors. 2 and 3 are prime, so circle them.	
Step 4. Write the composite number as the product of all the circled primes.		$48 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3$

We say $2 \cdot 2 \cdot 2 \cdot 2 \cdot 3$ is the prime factorization of 48. We generally write the primes in ascending order. Be sure to multiply the factors to verify your answer!

If we first factored 48 in a different way, for example as $6 \cdot 8$, the result would still be the same. Finish the prime factorization and verify this for yourself.

Note:
Exercise:

Problem: Find the prime factorization of 80.

Solution:

$2 \cdot 2 \cdot 2 \cdot 2 \cdot 5$

Note:
Exercise:

Problem: Find the prime factorization of 60.

Solution:

$2 \cdot 2 \cdot 3 \cdot 5$

Note:
Find the Prime Factorization of a Composite Number.

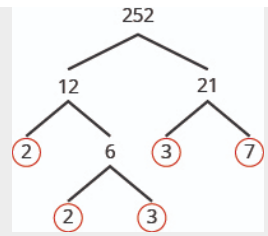
Find two factors whose product is the given number, and use these numbers to create two branches.
If a factor is prime, that branch is complete. Circle the prime, like a bud on the tree.
If a factor is not prime, write it as the product of two factors and continue the process.
Write the composite number as the product of all the circled primes.

Example:
Exercise:

Problem: Find the prime factorization of 252.

Solution:
Solution

Step 1. Find two factors whose product is 252. 12 and 21 are not prime.	

Break 12 and 21 into two more factors. Continue until all primes are factored.	
Step 2. Write 252 as the product of all the circled primes.	$252 = 2 \cdot 2 \cdot 3 \cdot 3 \cdot 7$

Note:
Exercise:

Problem: Find the prime factorization of 126.

Solution:

$2 \cdot 3 \cdot 3 \cdot 7$

Note:
Exercise:

Problem: Find the prime factorization of 294.

Solution:

$2 \cdot 3 \cdot 7 \cdot 7$

One of the reasons we look at multiples and primes is to use these techniques to find the **least common multiple** of two numbers. This will be useful when we add and subtract fractions with different denominators. Two methods are used most often to find the least common multiple and we will look at both of them.

The first method is the Listing Multiples Method. To find the least common multiple of 12 and 18, we list the first few multiples of 12 and 18:

12: 12, 24, **36**, 48, 60, **72**, 84, 96, **108**...

18: 18, **36**, 54, **72**, 90, **108**...

Common Multiples: 36, 72, 108...

Least Common Multiple: 36

Notice that some numbers appear in both lists. They are the **common multiples** of 12 and 18.

We see that the first few common multiples of 12 and 18 are 36, 72, and 108. Since 36 is the smallest of the common multiples, we call it the *least common multiple*. We often use the abbreviation LCM.

Note:
Least Common Multiple
The **least common multiple** (LCM) of two numbers is the smallest number that is a multiple of both numbers.

The procedure box lists the steps to take to find the LCM using the prime factors method we used above for 12 and 18.

Note:
Find the Least Common Multiple by Listing Multiples.

List several multiples of each number.
Look for the smallest number that appears on both lists.
This number is the LCM.

Example:
Exercise:

Problem: Find the least common multiple of 15 and 20 by listing multiples.

Solution:
Solution

Make lists of the first few multiples of 15 and of 20, and use them to find the least common multiple.	**15:** 15, 30, 45, **60**, 75, 90, 105, 120 **20:** 20, 40, **60**, 80, 100, 120, 140, 160
Look for the smallest number that appears in both lists.	The first number to appear on both lists is 60, so 60 is the least common multiple of 15 and 20.

Notice that 120 is in both lists, too. It is a common multiple, but it is not the *least* common multiple.

Note:
Exercise:

Problem: Find the least common multiple by listing multiples: 9 and 12.

Solution:

36

Note:
Exercise:

Problem: Find the least common multiple by listing multiples: 18 and 24.

Solution:

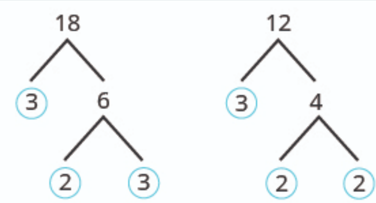
72

Our second method to find the least common multiple of two numbers is to use The Prime Factors Method. Let’s find the LCM of 12 and 18 again, this time using their prime factors.

Example:
How to Find the Least Common Multiple Using the Prime Factors Method
Exercise:

Problem: Find the Least Common Multiple (LCM) of 12 and 18 using the prime factors method.

Solution:
Solution

Step 1. Write each number as a product of primes.		
Step 2. List the primes of each number. Match primes vertically when possible.	List the primes of 12. List the primes of 18. Line up with the primes of 12 when possible. If not create a new column.	$12 = 2 \cdot 2 \cdot 3$ $18 = 2 \cdot \phantom{2 \cdot{}} 3 \cdot 3$
Step 3. Bring down the number from each column.		$12 = 2 \cdot 2 \cdot 3$ $18 = 2 \cdot \phantom{2 \cdot{}} 3 \cdot 3$ $\text{LCM} = 2 \cdot 2 \cdot 3 \cdot 3$
Step 4. Multiply the factors.		$\text{LCM} = 36$

Notice that the prime factors of 12 $(2 \cdot 2 \cdot 3)$ and the prime factors of 18 $(2 \cdot 3 \cdot 3)$ are included in the LCM $(2 \cdot 2 \cdot 3 \cdot 3)$. So 36 is the least common multiple of 12 and 18.

By matching up the common primes, each common prime factor is used only once. This way you are sure that 36 is the *least* common multiple.

Note:
Exercise:

Problem: Find the LCM using the prime factors method: 9 and 12.

Solution:

36

Note:
Exercise:

Problem: Find the LCM using the prime factors method: 18 and 24.

Solution:

72

Note:
Find the Least Common Multiple Using the Prime Factors Method.

Write each number as a product of primes.
List the primes of each number. Match primes vertically when possible.
Bring down the columns.
Multiply the factors.

Example:
Exercise:

Problem: Find the Least Common Multiple (LCM) of 24 and 36 using the prime factors method.

Solution:
Solution

Find the primes of 24 and 36. Match primes vertically when possible.	

Bring down all columns.	$24 = 2 \cdot 2 \cdot 2 \cdot 3$ $36 = 2 \cdot 2 \cdot \quad 3 \cdot 3$ $\text{LCM} = 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3$
Multiply the factors.	$\text{LCM} = 72$
	The LCM of 24 and 36 is 72.

Note:
Exercise:

Problem: Find the LCM using the prime factors method: 21 and 28.

Solution:

84

Note:
Exercise:

Problem: Find the LCM using the prime factors method: 24 and 32.

Solution:

96

Note:
Access this online resource for additional instruction and practice with using whole numbers. You will need to enable Java in your web browser to use the application.

- Sieve of Eratosthenes

Key Concepts

- **Place Value** as in [link].
- **Name a Whole Number in Words**

  Start at the left and name the number in each period, followed by the period name.
  Put commas in the number to separate the periods.
  Do not name the ones period.

- **Write a Whole Number Using Digits**

  Identify the words that indicate periods. (Remember the ones period is never named.)
  Draw 3 blanks to indicate the number of places needed in each period. Separate the periods by commas.
  Name the number in each period and place the digits in the correct place value position.

- **Round Whole Numbers**

  Locate the given place value and mark it with an arrow. All digits to the left of the arrow do not change.
  Underline the digit to the right of the given place value.
  Is this digit greater than or equal to 5?
    - Yes—add 1 to the digit in the given place value.
    - No—do <u>not</u> change the digit in the given place value.

  Replace all digits to the right of the given place value with zeros.

- **Divisibility Tests:** A number is divisible by:

  - 2 if the last digit is 0, 2, 4, 6, or 8.
  - 3 if the sum of the digits is divisible by 3.
  - 5 if the last digit is 5 or 0.
  - 6 if it is divisible by both 2 and 3.
  - 10 if it ends with 0.

- **Find the Prime Factorization of a Composite Number**

  Find two factors whose product is the given number, and use these numbers to create two branches.
  If a factor is prime, that branch is complete. Circle the prime, like a bud on the tree.
  If a factor is not prime, write it as the product of two factors and continue the process.
  Write the composite number as the product of all the circled primes.

- **Find the Least Common Multiple by Listing Multiples**

  List several multiples of each number.
  Look for the smallest number that appears on both lists.
  This number is the LCM.

- **Find the Least Common Multiple Using the Prime Factors Method**

  Write each number as a product of primes.
  List the primes of each number. Match primes vertically when possible.
  Bring down the columns.
  Multiply the factors.

Practice Makes Perfect

Use Place Value with Whole Numbers

In the following exercises, find the place value of each digit in the given numbers.
Exercise:

51,493
ⓐ 1
ⓑ 4
ⓒ 9
ⓓ 5
Problem: ⓔ 3

Solution:

ⓐ thousands ⓑ hundreds ⓒ tens ⓓ ten thousands ⓔ ones

Exercise:

87,210
ⓐ 2
ⓑ 8
ⓒ 0
ⓓ 7
Problem: ⓔ 1

Exercise:

164,285
ⓐ 5
ⓑ 6
ⓒ 1
ⓓ 8
Problem: ⓔ 2

Solution:

ⓐ ones ⓑ ten thousands ⓒ hundred thousands ⓓ tens ⓔ hundreds

Exercise:

395,076
ⓐ 5
ⓑ 3
ⓒ 7
ⓓ 0
Problem: ⓔ 9

Exercise:

93,285,170
ⓐ 9
ⓑ 8
ⓒ 7
ⓓ 5
Problem: ⓔ 3

Solution:

ⓐ ten millions ⓑ ten thousands ⓒ tens ⓓ thousands ⓔ millions

Exercise:

36,084,215
ⓐ 8
ⓑ 6
ⓒ 5
ⓓ 4
Problem: ⓔ 3

Exercise:

7,284,915,860,132
ⓐ 7
ⓑ 4
ⓒ 5
ⓓ 3
Problem: ⓔ 0

Solution:

ⓐ trillions ⓑ billions ⓒ millions ⓓ tens ⓔ thousands

Exercise:

2,850,361,159,433
ⓐ 9
ⓑ 8
ⓒ 6
ⓓ 4
Problem: ⓔ 2

In the following exercises, name each number using words.

Exercise:

Problem: 1,078

Solution:

one thousand, seventy-eight

Exercise:

Problem: 5,902

Exercise:

Problem: 364,510

Solution:

three hundred sixty-four thousand, five hundred ten

Exercise:

Problem: 146,023

Exercise:

Problem: 5,846,103

Solution:

five million, eight hundred forty-six thousand, one hundred three

Exercise:

Problem: 1,458,398

Exercise:

Problem: 37,889,005

Solution:

thirty-seven million, eight hundred eighty-nine thousand, five

Exercise:

Problem: 62,008,465

In the following exercises, write each number as a whole number using digits.

Exercise:

Problem: four hundred twelve

Solution:

412

Exercise:

Problem: two hundred fifty-three

Exercise:

Problem: thirty-five thousand, nine hundred seventy-five

Solution:

35,975

Exercise:

Problem: sixty-one thousand, four hundred fifteen

Exercise:

Problem: eleven million, forty-four thousand, one hundred sixty-seven

---

Solution:

11,044,167

Exercise:

Problem: eighteen million, one hundred two thousand, seven hundred eighty-three

Exercise:

Problem: three billion, two hundred twenty-six million, five hundred twelve thousand, seventeen

Solution:

3,226,512,017

Exercise:

Problem: eleven billion, four hundred seventy-one million, thirty-six thousand, one hundred six

In the following, round to the indicated place value.

Exercise:

Problem: Round to the nearest ten.

ⓐ 386 ⓑ 2,931

Solution:

ⓐ 390 ⓑ 2,930

Exercise:

Problem: Round to the nearest ten.

ⓐ 792 ⓑ 5,647

Exercise:

Problem: Round to the nearest hundred.

ⓐ 13,748 ⓑ 391,794

Solution:

ⓐ 13,700 ⓑ 391,800

Exercise:

Problem: Round to the nearest hundred.

ⓐ 28,166 ⓑ 481,628

Exercise:

Problem: Round to the nearest ten.

ⓐ 1,492 ⓑ 1,497

Solution:

ⓐ 1,490 ⓑ 1,500

Exercise:

Problem: Round to the nearest ten.

ⓐ 2,791 ⓑ 2,795

Exercise:

Problem: Round to the nearest hundred.

ⓐ 63,994 ⓑ 63,040

Solution:

ⓐ 64,000 ⓑ 63,900

Exercise:

Problem: Round to the nearest hundred.

ⓐ 49,584 ⓑ 49,548

In the following exercises, round each number to the nearest ⓐ hundred, ⓑ thousand, ⓒ ten thousand.

Exercise:

Problem: 392,546

Solution:

ⓐ 392,500 ⓑ 393,000 ⓒ 390,000

Exercise:

Problem: 619,348

Exercise:

Problem: 2,586,991

Solution:

ⓐ 2,587,000 ⓑ 2,587,000 ⓒ 2,590,000

Exercise:

Problem: 4,287,965

Identify Multiples and Factors

In the following exercises, use the divisibility tests to determine whether each number is divisible by 2, 3, 5, 6, and 10.

Exercise:

Problem: 84

Solution:

divisible by 2, 3, and 6

Exercise:

Problem: 9,696

Exercise:

Problem: 75

Solution:

divisible by 3 and 5

Exercise:

Problem: 78

Exercise:

Problem: 900

Solution:

divisible by 2, 3, 5, 6, and 10

Exercise:

Problem: 800

Exercise:

Problem: 986

Solution:

divisible by 2

Exercise:

Problem: 942

Exercise:

Problem: 350

Solution:

divisible by 2, 3, and 10

Exercise:

Problem: 550

Exercise:

Problem: 22,335

Solution:

divisible by 3 and 5

Exercise:

Problem: 39,075

Find Prime Factorizations and Least Common Multiples

In the following exercises, find the prime factorization.

Exercise:

Problem: 86

Solution:

$2 \cdot 43$

Exercise:

Problem: 78

Exercise:

Problem: 132

Solution:

$2 \cdot 2 \cdot 3 \cdot 11$

Exercise:

Problem: 455

Exercise:

Problem: 693

Solution:

$3 \cdot 3 \cdot 7 \cdot 11$

Exercise:

Problem: 400

Exercise:

Problem: 432

Solution:

$2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3$

Exercise:

Problem: 627

Exercise:

Problem: 2,160

Solution:

$2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3 \cdot 5$

Exercise:

Problem: 2,520

In the following exercises, find the least common multiple of the each pair of numbers using the multiples method.

Exercise:

Problem: 8, 12

Solution:

24

Exercise:

Problem: 4, 3

Exercise:

Problem: 12, 16

Solution:

48

Exercise:

Problem: 30, 40

Exercise:

Problem: 20, 30

Solution:

60

Exercise:

Problem: 44, 55

In the following exercises, find the least common multiple of each pair of numbers using the prime factors method.

Exercise:

Problem: 8, 12

Solution:

24

Exercise:

Problem: 12, 16

Exercise:

Problem: 28, 40

Solution:

420

Exercise:

Problem: 84, 90

Exercise:

Problem: 55, 88

Solution:

440

Exercise:

Problem: 60, 72

Everyday Math

Exercise:

Problem:

Writing a Check Jorge bought a car for $24,493. He paid for the car with a check. Write the purchase price in words.

Solution:

twenty-four thousand, four hundred ninety-three dollars

Exercise:

Problem:

Writing a Check Marissa’s kitchen remodeling cost $18,549. She wrote a check to the contractor. Write the amount paid in words.

Exercise:

Problem:

Buying a Car Jorge bought a car for $24,493. Round the price to the nearest ⓐ ten ⓑ hundred ⓒ thousand; and ⓓ ten-thousand.

Solution:

ⓐ $24,490 ⓑ $24,500 ⓒ $24,000 ⓓ $20,000

Exercise:

Problem:

Remodeling a Kitchen Marissa’s kitchen remodeling cost $18,549, Round the cost to the nearest ⓐ ten ⓑ hundred ⓒ thousand and ⓓ ten-thousand.

Exercise:

Problem:

Population The population of China was 1,339,724,852 on November 1, 2010. Round the population to the nearest ⓐ billion ⓑ hundred-million; and ⓒ million.

Solution:

ⓐ 1,000,000,000 ⓑ 1,300,000,000 ⓒ 1,340,000,000

Exercise:

Problem:

Astronomy The average distance between Earth and the sun is 149,597,888 kilometers. Round the distance to the nearest ⓐ hundred-million ⓑ ten-million; and ⓒ million.

Exercise:

Problem:

Grocery Shopping Hot dogs are sold in packages of 10, but hot dog buns come in packs of eight. What is the smallest number that makes the hot dogs and buns come out even?

Solution:

80

Exercise:

Problem:

Grocery Shopping Paper plates are sold in packages of 12 and party cups come in packs of eight. What is the smallest number that makes the plates and cups come out even?

Writing Exercises

Exercise:

Problem: Give an everyday example where it helps to round numbers.

Exercise:

Problem: If a number is divisible by 2 and by 3 why is it also divisible by 6?

Exercise:

Problem: What is the difference between prime numbers and composite numbers?

Solution:

Answers may vary.

Exercise:

Problem:

Explain in your own words how to find the prime factorization of a composite number, using any method you prefer.

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can…	Confidently	With some help	No-I don't get it!
use place value with whole numbers.			
identify multiples and apply divisibility tests.			
find prime factorizations and least common multiples.			

ⓑ If most of your checks were:

…confidently. Congratulations! You have achieved the objectives in this section. Reflect on the study skills you used so that you can continue to use them. What did you do to become confident of your ability to do these things? Be specific.

…with some help. This must be addressed quickly because topics you do not master become potholes in your road to success. In math, every topic builds upon previous work. It is important to make sure you have a strong foundation before you move on. Who can you ask for help? Your fellow classmates and instructor are good resources. Is there a place on campus where math tutors are available? Can your study skills be improved?

…no—I don't get it! This is a warning sign and you must not ignore it. You should get help right away or you will quickly be overwhelmed. See your instructor as soon as you can to discuss your situation. Together you can come up with a plan to get you the help you need.

Glossary

composite number
: A composite number is a counting number that is not prime. A composite number has factors other than 1 and itself.

counting numbers
: The counting numbers are the numbers 1, 2, 3, …

divisible by a number

If a number m is a multiple of n, then m is divisible by n. (If 6 is a multiple of 3, then 6 is divisible by 3.)

factors
: If $a \cdot b = m$, then a and b are factors of m. Since 3 · 4 = 12, then 3 and 4 are factors of 12.

least common multiple
: The least common multiple of two numbers is the smallest number that is a multiple of both numbers.

multiple of a number
: A number is a multiple of *n* if it is the product of a counting number and *n*.

number line
: A number line is used to visualize numbers. The numbers on the number line get larger as they go from left to right, and smaller as they go from right to left.

origin
: The origin is the point labeled 0 on a number line.

prime factorization
: The prime factorization of a number is the product of prime numbers that equals the number.

prime number
: A prime number is a counting number greater than 1, whose only factors are 1 and itself.

whole numbers
: The whole numbers are the numbers 0, 1, 2, 3, ...

Use the Language of Algebra
By the end of this section, you will be able to:

- Use variables and algebraic symbols
- Simplify expressions using the order of operations
- Evaluate an expression
- Identify and combine like terms
- Translate an English phrase to an algebraic expression

Note:
A more thorough introduction to the topics covered in this section can be found in the *Prealgebra* chapter, **The Language of Algebra**.

Use Variables and Algebraic Symbols

Suppose this year Greg is 20 years old and Alex is 23. You know that Alex is 3 years older than Greg. When Greg was 12, Alex was 15. When Greg is 35, Alex will be 38. No matter what Greg's age is, Alex's age will always be 3 years more, right? In the language of algebra, we say that Greg's age and Alex's age are **variables** and the 3 is a **constant**. The ages change ("vary") but the 3 years between them always stays the same ("constant"). Since Greg's age and Alex's age will always differ by 3 years, 3 is the *constant*.

In algebra, we use letters of the alphabet to represent variables. So if we call Greg's age *g*, then we could use $g+3$ to represent Alex's age. See [link].

Greg's age	**Alex's age**
12	15
20	23
35	38
$\boldsymbol{g}$	$\boldsymbol{g}+3$

The letters used to represent these changing ages are called *variables*. The letters most commonly used for variables are *x*, *y*, *a*, *b*, and *c*.

Note:
Variable
A **variable** is a letter that represents a number whose value may change.

Note:
Constant
A **constant** is a number whose value always stays the same.

To write algebraically, we need some operation symbols as well as numbers and variables. There are several types of symbols we will be using.

There are four basic arithmetic operations: addition, subtraction, multiplication, and division. We'll list the symbols used to indicate these operations below ([link]). You'll probably recognize some of them.

Operation	**Notation**	**Say:**	**The result is…**
Addition	$a+b$	*a* plus *b*	the sum of *a* and *b*
Subtraction	$a-b$	*a* minus *b*	the difference of *a* and *b*
Multiplication	$a\cdot b, ab, (a)\,(b),$ $(a)b, a\,(b)$	*a* times *b*	the product of *a* and *b*
Division	$a\div b, a/b, \frac{a}{b}, b\overline{)a}$	*a* divided by *b*	the quotient of *a* and *b*, *a* is called the dividend, and *b* is called the divisor

We perform these operations on two numbers. When translating from symbolic form to English, or from English to symbolic form, pay attention to the words "of" and "and."

- The *difference of* 9 and 2 means subtract 9 and 2, in other words, 9 minus 2, which we write symbolically as $9-2.$
- The *product of* 4 and 8 means multiply 4 and 8, in other words 4 times 8, which we write symbolically as $4\cdot 8.$

In algebra, the cross symbol, $\times$, is not used to show multiplication because that symbol may cause confusion. Does 3*xy* mean $3\ \times\ y$ ('three times *y*') or $3\cdot x\cdot y$ (three times *x* times *y*)? To make it clear, use · or parentheses for multiplication.

When two quantities have the same value, we say they are equal and connect them with an **equal sign**.

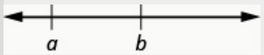
Note:
Equality Symbol
$a=b$ is read "*a* is equal to *b*"
The symbol "$=$" is called the **equal sign**.

On the number line, the numbers get larger as they go from left to right. The number line can be used to explain the symbols "<" and ">."

Note:
Inequality

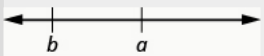
Equation:

$$a<b \text{ is read "}a\text{ is less than }b\text{"}$$
$$a \text{ is to the left of } b \text{ on the number line}$$



Equation:

$$a>b \text{ is read "}a\text{ is greater than }b\text{"}$$
$$a \text{ is to the right of } b \text{ on the number line}$$



The expressions *a* < b or a > *b* can be read from left to right or right to left, though in English we usually read from left to right ([link]). In general, *a* < b is equivalent to b > *a*. For example 7 < 11 is equivalent to 11 > 7. And *a* > *b* is equivalent to *b* < a. For example 17 > 4 is equivalent to 4 < 17.

Inequality Symbols	Words
$a \neq b$	*a* is *not equal to b*
a < *b*	*a* is *less than b*
$a \leq b$	*a* is *less than or equal to b*
a > *b*	*a* is *greater than b*
$a \geq b$	*a* is *greater than or equal to b*

Example:
Exercise:

Problem: Translate from algebra into English:

ⓐ $17 \leq 26$ ⓑ $8 \neq 17-3$ ⓒ $12>27 \div 3$ ⓓ $y+7<19$

Solution:
Solution

ⓐ $17 \leq 26$
17 is less than or equal to 26

ⓑ $8 \neq 17 - 3$
8 is not equal to 17 minus 3

ⓒ $12 > 27 \div 3$
12 is greater than 27 divided by 3

ⓓ $y + 7 < 19$
y plus 7 is less than 19

Note:
Exercise:

Problem: Translate from algebra into English:

ⓐ $14 \leq 27$ ⓑ $19 - 2 \neq 8$ ⓒ $12 > 4 \div 2$ ⓓ $x - 7 < 1$

Solution:

ⓐ 14 is less than or equal to 27 ⓑ 19 minus 2 is not equal to 8 ⓒ 12 is greater than 4 divided by 2 ⓓ *x* minus 7 is less than 1

Note:
Exercise:

Problem: Translate from algebra into English:

ⓐ $19 \geq 15$ ⓑ $7 = 12 - 5$ ⓒ $15 \div 3 < 8$ ⓓ $y + 3 > 6$

Solution:

ⓐ 19 is greater than or equal to 15 ⓑ 7 is equal to 12 minus 5 ⓒ 15 divided by 3 is less than 8 ⓓ *y* plus 3 is greater than 6

Grouping symbols in algebra are much like the commas, colons, and other punctuation marks in English. They help to make clear which expressions are to be kept together and separate from other expressions. We will introduce three types now.

Note:
Grouping Symbols
Equation:

$$\begin{array}{ll} \text{Parentheses} & () \\ \text{Brackets} & [] \\ \text{Braces} & \{\} \end{array}$$

Here are some examples of expressions that include grouping symbols. We will simplify expressions like these later in this section.
Equation:

$$8(14-8) \qquad 21-3[2+4(9-8)] \qquad 24 \div \{13-2[1(6-5)+4]\}$$

What is the difference in English between a phrase and a sentence? A phrase expresses a single thought that is incomplete by itself, but a sentence makes a complete statement. “Running very fast” is a phrase, but “The football player was running very fast” is a sentence. A sentence has a subject and a verb. In algebra, we have *expressions* and *equations*.

Note:
Expression
An **expression** is a number, a variable, or a combination of numbers and variables using operation symbols.

An **expression** is like an English phrase. Here are some examples of expressions:

Expression	**Words**	**English Phrase**
$3+5$	3 plus 5	the sum of three and five
$n-1$	*n* minus one	the difference of *n* and one
$6 \cdot 7$	6 times 7	the product of six and seven
$\frac{x}{y}$	*x* divided by *y*	the quotient of *x* and *y*

Notice that the English phrases do not form a complete sentence because the phrase does not have a verb.

An **equation** is two expressions linked with an equal sign. When you read the words the symbols represent in an equation, you have a complete sentence in English. The equal sign gives the verb.

Note:
Equation
An **equation** is two expressions connected by an equal sign.

Here are some examples of equations.

Equation	English Sentence
$3+5=8$	The sum of three and five is equal to eight.
$n-1=14$	*n* minus one equals fourteen.
$6 \cdot 7=42$	The product of six and seven is equal to forty-two.
$x=53$	*x* is equal to fifty-three.
$y+9=2y-3$	*y* plus nine is equal to two *y* minus three.

Example:
Exercise:

Problem: Determine if each is an expression or an equation:

ⓐ $2(x+3)=10$ ⓑ $4(y-1)+1$ ⓒ $x \div 25$ ⓓ $y+8=40$

Solution:
Solution

ⓐ $2(x+3)=10$ This is an *equation*—two expressions are connected with an equal sign.
ⓑ $4(y-1)+1$ This is an *expression*—no equal sign.
ⓒ $x \div 25$ This is an *expression*—no equal sign.
ⓓ $y+8=40$ This is an *equation*—two expressions are connected with an equal sign.

Note:
Exercise:

Problem: Determine if each is an expression or an equation: ⓐ $3(x-7)=27$ ⓑ $5(4y-2)-7$.

Solution:

ⓐ equation ⓑ expression

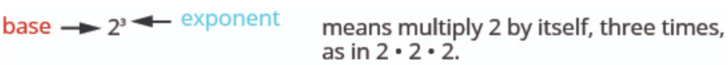
Note:
Exercise:

Problem: Determine if each is an expression or an equation: ⓐ $y^3 \div 14$ ⓑ $4x-6=22$.

Solution:

ⓐ expression ⓑ equation

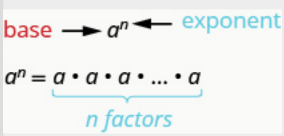
Suppose we need to multiply 2 nine times. We could write this as $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$. This is tedious and it can be hard to keep track of all those 2s, so we use exponents. We write $2 \cdot 2 \cdot 2$ as 2^3 and $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$ as 2^9. In expressions such as 2^3, the 2 is called the *base* and the 3 is called the *exponent*. The exponent tells us how many times we need to multiply the base.



We read 2^3 as "two to the third power" or "two cubed."

We say 2^3 is in *exponential notation* and $2 \cdot 2 \cdot 2$ is in *expanded notation*.

Note:
Exponential Notation
a^n means multiply *a* by itself, *n* times.



The expression a^n is read *a* to the n^{th} power.

While we read a^n as "*a* to the n^{th} power," we usually read:

- a^2 "*a* squared"
- a^3 "*a* cubed"

We'll see later why a^2 and a^3 have special names.

[link] shows how we read some expressions with exponents.

Expression	In Words
7^2	7 to the second power or 7 squared
5^3	5 to the third power or 5 cubed
9^4	9 to the fourth power
12^5	12 to the fifth power

Example:
Exercise:

Problem: Simplify: 3^4.

Solution:
Solution

	3^4
Expand the expression.	$3 \cdot 3 \cdot 3 \cdot 3$
Multiply left to right.	$9 \cdot 3 \cdot 3$
Multiply.	$27 \cdot 3$
Multiply.	81

Note:
Exercise:

Problem: Simplify: ⓐ 5^3 ⓑ 1^7.

Solution:

ⓐ 125 ⓑ 1

Note:
Exercise:

Problem: Simplify: ⓐ 7^2 ⓑ 0^5.

Solution:

ⓐ 49 ⓑ 0

Simplify Expressions Using the Order of Operations

To **simplify an expression** means to do all the math possible. For example, to simplify $4 \cdot 2 + 1$ we'd first multiply $4 \cdot 2$ to get 8 and then add the 1 to get 9. A good habit to develop is to work down the page, writing each step of the process below the previous step. The example just described would look like this:
Equation:

$$\begin{array}{c} 4 \cdot 2 + 1 \\ 8 + 1 \\ 9 \end{array}$$

By not using an equal sign when you simplify an expression, you may avoid confusing expressions with equations.

Note:
Simplify an Expression
To **simplify an expression**, do all operations in the expression.

We've introduced most of the symbols and notation used in algebra, but now we need to clarify the order of operations. Otherwise, expressions may have different meanings, and they may result in different values. For example, consider the expression:
Equation:

$$4+3\cdot 7$$

If you simplify this expression, what do you get?

Some students say 49,
Equation:

$$\begin{array}{ll} & 4+3\cdot 7 \\ \text{Since } 4+3 \text{ gives } 7. & 7\cdot 7 \\ \text{And } 7\cdot 7 \text{ is } 49. & 49 \end{array}$$

Others say 25,
Equation:

$$\begin{array}{ll} & 4+3\cdot 7 \\ \text{Since } 3\cdot 7 \text{ is } 21. & 4+21 \\ \text{And } 21+4 \text{ makes } 25. & 25 \end{array}$$

Imagine the confusion in our banking system if every problem had several different correct answers!

The same expression should give the same result. So mathematicians early on established some guidelines that are called the Order of Operations.

Note:
Perform the Order of Operations.

Parentheses and Other Grouping Symbols

- Simplify all expressions inside the parentheses or other grouping symbols, working on the innermost parentheses first.

Exponents

- Simplify all expressions with exponents.

Multiplication and Division

- Perform all multiplication and division in order from left to right. These operations have equal priority.

Addition and **S**ubtraction

- Perform all addition and subtraction in order from left to right. These operations have equal priority.

Note: Doing the Manipulative Mathematics activity "Game of 24" give you practice using the order of operations.

Students often ask, “How will I remember the order?” Here is a way to help you remember: Take the first letter of each key word and substitute the silly phrase: “Please Excuse My Dear Aunt Sally.”
Equation:

Parentheses	**P**lease
Exponents	**E**xcuse
Multiplication **D**ivision	**M**y **D**ear
Addition **S**ubtraction	**A**unt **S**ally

It’s good that “**M**y **D**ear” goes together, as this reminds us that **m**ultiplication and **d**ivision have equal priority. We do not always do multiplication before division or always do division before multiplication. We do them in order from left to right.

Similarly, “**A**unt **S**ally” goes together and so reminds us that **a**ddition and **s**ubtraction also have equal priority and we do them in order from left to right.

Let’s try an example.

Example:
Exercise:

Problem: Simplify: ⓐ $4+3\cdot 7$ ⓑ $(4+3)\cdot 7$.

Solution:
Solution

ⓐ

	$4+3\cdot 7$
Are there any **p**arentheses? No.	
Are there any **e**xponents? No.	
Is there any **m**ultiplication or **d**ivision? Yes.	
Multiply first.	$4+3\cdot 7$
Add.	$4+21$
	25

ⓑ

	$(4+3) \cdot 7$
Are there any **p**arentheses? Yes.	$(4+3) \cdot 7$
Simplify inside the parentheses.	$(7)7$
Are there any **e**xponents? No.	
Is there any **m**ultiplication or **d**ivision? Yes.	
Multiply.	49

Note:
Exercise:

Problem: Simplify: ⓐ $12 - 5 \cdot 2$ ⓑ $(12 - 5) \cdot 2$.

Solution:

ⓐ 2 ⓑ 14

Note:
Exercise:

Problem: Simplify: ⓐ $8 + 3 \cdot 9$ ⓑ $(8 + 3) \cdot 9$.

Solution:

ⓐ 35 ⓑ 99

Example:
Exercise:

Problem: Simplify: $18 \div 6 + 4\,(5 - 2)$.

Solution:
Solution

Parentheses? Yes, subtract first.	$18 \div 6 + 4\,(5 - 2)$ $18 \div 6 + 4(3)$
Exponents? No.	
Multiplication or division? Yes.	$18 \div 6 + 4(3)$
Divide first because we multiply and divide left to right.	$3 + 4(3)$
Any other multiplication or division? Yes.	
Multiply.	$3 + 12$
Any other multiplication or division? No.	
Any addition or subtraction? Yes.	15

Note:
Exercise:

Problem: Simplify: $30 \div 5 + 10\,(3 - 2)$.

Solution:

16

Note:
Exercise:

Problem: Simplify: $70 \div 10 + 4\,(6 - 2)$.

Solution:

23

When there are multiple grouping symbols, we simplify the innermost parentheses first and work outward.

Example:
Exercise:

Problem: Simplify: $5 + 2^3 + 3\left[6 - 3\left(4 - 2\right)\right]$.

Solution:
Solution

	$5 + 2^3 + 3[6 - 3(4 - 2)]$
Are there any parentheses (or other grouping symbol)? Yes.	
Focus on the parentheses that are inside the brackets.	$5 + 2^3 + 3[6 - 3(4 - 2)]$
Subtract.	$5 + 2^3 + 3[6 - 3(2)]$
Continue inside the brackets and multiply.	$5 + 2^3 + 3[6 - 6]$
Continue inside the brackets and subtract.	$5 + 2^3 + 3[0]$
The expression inside the brackets requires no further simplification.	
Are there any exponents? Yes.	$5 + 2^3 + 3[0]$
Simplify exponents.	$5 + 8 + 3[0]$
Is there any multiplication or division? Yes.	
Multiply.	$5 + 8 + 0$
Is there any addition or subtraction? Yes.	

Add.	$13+0$
Add.	13

Note:
Exercise:

Problem: Simplify: $9+5^3-[4(9+3)]$.

Solution:

86

Note:
Exercise:

Problem: Simplify: $7^2-2[4(5+1)]$.

Solution:

1

Evaluate an Expression

In the last few examples, we simplified expressions using the order of operations. Now we'll evaluate some expressions—again following the order of operations. To **evaluate an expression** means to find the value of the expression when the variable is replaced by a given number.

Note:
Evaluate an Expression
To **evaluate an expression** means to find the value of the expression when the variable is replaced by a given number.

To evaluate an expression, substitute that number for the variable in the expression and then simplify the expression.

Example:
Exercise:

Problem: Evaluate $7x-4$, when ⓐ $x=5$ and ⓑ $x=1$.

Solution:
Solution

ⓐ

when $x = 5$	$7x - 4$
	$7(5) - 4$
Multiply.	$35 - 4$
Subtract.	31

ⓑ

when $x = 1$	$7x - 4$
	$7(1) - 4$
Multiply.	$7 - 4$
Subtract.	3

Note:
Exercise:

Problem: Evaluate $8x - 3$, when ⓐ $x = 2$ and ⓑ $x = 1$.

Solution:

ⓐ 13 ⓑ 5

Note:
Exercise:

Problem: Evaluate $4y - 4$, when ⓐ $y = 3$ and ⓑ $y = 5$.

Solution:

ⓐ 8 ⓑ 16

Example:
Exercise:

Problem: Evaluate $x = 4$, when ⓐ x^2 ⓑ 3^x.

Solution:
Solution

ⓐ

	x^2
Replace *x* with 4.	4^2
Use definition of exponent.	$4 \cdot 4$
Simplify.	16

ⓑ

	3^x
Replace *x* with 4.	3^4
Use definition of exponent.	$3 \cdot 3 \cdot 3 \cdot 3$
Simplify.	81

Note:
Exercise:

Problem: Evaluate $x = 3$, when ⓐ x^2 ⓑ 4^x.

Solution:

ⓐ 9 ⓑ 64

Note:
Exercise:

Problem: Evaluate $x = 6$, when ⓐ x^3 ⓑ 2^x.

Solution:

ⓐ 216 ⓑ 64

Example:
Exercise:

Problem: Evaluate $2x^2 + 3x + 8$ when $x = 4$.

Solution:
Solution

	$2x^2 + 3x + 8$
Substitute $x = 4$.	$2(4)^2 + 3(4) + 8$
Follow the order of operations.	$2(16) + 3(4) + 8$
	$32 + 12 + 8$
	52

Note:

Exercise:

Problem: Evaluate $3x^2 + 4x + 1$ when $x = 3$.

Solution:

40

Note:
Exercise:

Problem: Evaluate $6x^2 - 4x - 7$ when $x = 2$.

Solution:

9

Indentify and Combine Like Terms

Algebraic expressions are made up of terms. A **term** is a constant, or the product of a constant and one or more variables.

Note:
Term
A **term** is a constant, or the product of a constant and one or more variables.

Examples of terms are $7, y, 5x^2, 9a, \text{and } b^5$.

The constant that multiplies the variable is called the **coefficient**.

Note:
Coefficient
The **coefficient** of a term is the constant that multiplies the variable in a term.

Think of the coefficient as the number in front of the variable. The coefficient of the term 3*x* is 3. When we write *x*, the coefficient is 1, since $x = 1 \cdot x$.

Example:
Exercise:

Problem: Identify the coefficient of each term: ⓐ $14y$ ⓑ $15x^2$ ⓒ a.

Solution:
Solution

ⓐ The coefficient of $14y$ is 14.

ⓑ The coefficient of $15x^2$ is 15.

ⓒ The coefficient of a is 1 since $a = 1\ a$.

Note:
Exercise:

Problem: Identify the coefficient of each term: ⓐ $17x$ ⓑ $41b^2$ ⓒ z.

Solution:

ⓐ 14 ⓑ 41 ⓒ 1

Note:
Exercise:

Problem: Identify the coefficient of each term: ⓐ $9p$ ⓑ $13a^3$ ⓒ y^3.

Solution:

ⓐ 9 ⓑ 13 ⓒ 1

Some terms share common traits. Look at the following 6 terms. Which ones seem to have traits in common?
Equation:

$$5x \qquad 7 \qquad n^2 \qquad 4 \qquad 3x \qquad 9n^2$$

The 7 and the 4 are both constant terms.

The $5x$ and the $3x$ are both terms with x.

The n^2 and the $9n^2$ are both terms with n^2.

When two terms are constants or have the same variable and exponent, we say they are **like terms**.

- 7 and 4 are like terms.
- $5x$ and $3x$ are like terms.
- x^2 and $9x^2$ are like terms.

Note:
Like Terms
Terms that are either constants or have the same variables raised to the same powers are called **like terms**.

Example:
Exercise:

Problem: Identify the like terms: y^3, $7x^2$, 14, 23, $4y^3$, 9*x*, $5x^2$.

Solution:
Solution

y^3 and $4y^3$ are like terms because both have y^3; the variable and the exponent match.

$7x^2$ and $5x^2$ are like terms because both have x^2; the variable and the exponent match.

14 and 23 are like terms because both are constants.

There is no other term like 9*x*.

Note:
Exercise:

Problem: Identify the like terms: 9, $2x^3$, y^2, $8x^3$, 15, $9y$, $11y^2$.

Solution:

9 and 15, y^2 and $11y^2$, $2x^3$ and $8x^3$

Note:
Exercise:

Problem: Identify the like terms: $4x^3$, $8x^2$, 19, $3x^2$, 24, $6x^3$.

Solution:

19 and 24, $8x^2$ and $3x^2$, $4x^3$ and $6x^3$

Adding or subtracting terms forms an expression. In the expression $2x^2 + 3x + 8$, from [link], the three terms are $2x^2$, $3x$, and 8.

Example:
Exercise:

Problem: Identify the terms in each expression.

ⓐ $9x^2 + 7x + 12$
ⓑ $8x + 3y$

Solution:
Solution

ⓐ The terms of $9x^2 + 7x + 12$ are $9x^2$, 7*x*, and 12.

ⓑ The terms of $8x + 3y$ are 8*x* and 3*y*.

Note:
Exercise:

Problem: Identify the terms in the expression $4x^2 + 5x + 17$.

Solution:

$4x^2, 5x, 17$

Note:
Exercise:

Problem: Identify the terms in the expression $5x + 2y$.

Solution:

5*x*, 2*y*

If there are like terms in an expression, you can simplify the expression by combining the like terms. What do you think $4x + 7x + x$ would simplify to? If you thought 12*x*, you would be right!
Equation:

$$\begin{gathered} 4x + 7x + x \\ x + x + x + x \quad + x + x + x + x + x + x + x \quad + x \\ 12x \end{gathered}$$

Add the coefficients and keep the same variable. It doesn't matter what *x* is—if you have 4 of something and add 7 more of the same thing and then add 1 more, the result is 12 of them. For example, 4 oranges plus 7 oranges plus 1 orange is 12 oranges. We will discuss the mathematical properties behind this later.

Simplify: $4x + 7x + x$.

Add the coefficients. 12*x*

Example:

How To Combine Like Terms
Exercise:

Problem: Simplify: $2x^2 + 3x + 7 + x^2 + 4x + 5$.

Solution:
Solution

Step 1. Identify the like terms.	$2x^2 + 3x + 7 + x^2 + 4x + 5$ $2x^2 + 3x + 7 + x^2 + 4x + 5$
Step 2. Rearrange the expression so the like terms are together.	$2x^2 + x^2 + 3x + 4x + 7 + 5$
Step 3. Combine like terms.	$3x^2 + 7x + 12$

Note:
Exercise:

Problem: Simplify: $3x^2 + 7x + 9 + 7x^2 + 9x + 8$.

Solution:

$10x^2 + 16x + 17$

Note:
Exercise:

Problem: Simplify: $4y^2 + 5y + 2 + 8y^2 + 4y + 5$.

Solution:

$12y^2 + 9y + 7$

Note:
Combine Like Terms.

Identify like terms.
Rearrange the expression so like terms are together.
Add or subtract the coefficients and keep the same variable for each group of like terms.

Translate an English Phrase to an Algebraic Expression

In the last section, we listed many operation symbols that are used in algebra, then we translated expressions and equations into English phrases and sentences. Now we’ll reverse the process. We’ll translate English phrases into algebraic expressions. The symbols and variables we’ve talked about will help us do that. [link] summarizes them.

Operation	Phrase	Expression
Addition	a plus b the sum of a and b a increased by b b more than a the total of a and b b added to a	$a+b$
Subtraction	a minus b the difference of a and b a decreased by b b less than a b subtracted from a	$a-b$
Multiplication	a times b the product of a and b twice a	$a\cdot b, ab, a(b), (a)(b)$ $2a$
Division	a divided by b the quotient of a and b the ratio of a and b b divided into a	$a\div b, a/b, \frac{a}{b}, b\overline{)a}$

Look closely at these phrases using the four operations:

the **sum** *of a and b*

the **difference** *of a and b*

the **product** *of a and b*

the **quotient** *of a and b*

Each phrase tells us to operate on two numbers. Look for the words *of* and *and* to find the numbers.

Example:
Exercise:

Problem:

Translate each English phrase into an algebraic expression: ⓐ the difference of $17x$ and 5 ⓑ the quotient of $10x^2$ and 7.

Solution:
Solution

ⓐ The key word is *difference*, which tells us the operation is subtraction. Look for the words *of* and *and* to find the numbers to subtract.

the *difference of* 17*x and* 5

17*x* minus 5

17*x* – 5

ⓑ The key word is “quotient,” which tells us the operation is division.

the *quotient of* 10*x*² *and* 7

divide 10*x*² by 7

10*x*² ÷ 7

This can also be written $10x^2/7$ or $\frac{10x^2}{7}$.

Note:
Exercise:

Problem:

Translate the English phrase into an algebraic expression: ⓐ the difference of $14x^2$ and 13 ⓑ the quotient of 12*x* and 2.

Solution:

ⓐ $14x^2 - 13$ ⓑ $12x \div 2$

Note:
Exercise:

Problem:

Translate the English phrase into an algebraic expression: ⓐ the sum of $17y^2$ and 19 ⓑ the product of 7 and y.

Solution:

ⓐ $17y^2 + 19$ ⓑ 7*y*

How old will you be in eight years? What age is eight more years than your age now? Did you add 8 to your present age? Eight “more than” means 8 added to your present age. How old were you seven years ago? This is 7 years less than your age now. You subtract 7 from your present age. Seven “less than” means 7 subtracted from your present age.

Example:

Exercise:

Problem:

Translate the English phrase into an algebraic expression: ⓐ Seventeen more than *y* ⓑ Nine less than $9x^2$.

Solution:

Solution

ⓐ The key words are *more than.* They tell us the operation is addition. *More than* means "added to."

Equation:

$$\begin{array}{c}\text{Seventeen more than } y\\ \text{Seventeen added to } y\\ y+17\end{array}$$

ⓑ The key words are *less than*. They tell us to subtract. *Less than* means "subtracted from."

Equation:

$$\begin{array}{c}\text{Nine less than } 9x^2\\ \text{Nine subtracted from } 9x^2\\ 9x^2-9\end{array}$$

Note:

Exercise:

Problem:

Translate the English phrase into an algebraic expression: ⓐ Eleven more than x ⓑ Fourteen less than $11a$.

Solution:

ⓐ $x+11$ ⓑ $11a-14$

Note:

Exercise:

Problem: Translate the English phrase into an algebraic expression: ⓐ 13 more than *z* ⓑ 18 less than 8*x*.

Solution:

ⓐ $z+13$ ⓑ $8x-18$

Example:

Exercise:

Problem:

Translate the English phrase into an algebraic expression: ⓐ five times the sum of *m* and *n* ⓑ the sum of five times *m* and *n*.

Solution:
Solution

There are two operation words—*times* tells us to multiply and *sum* tells us to add.

ⓐ Because we are multiplying 5 times the sum we need parentheses around the sum of *m* and *n*, $(m+n)$. This forces us to determine the sum first. (Remember the order of operations.)
Equation:

$$\begin{array}{c}\text{five times the sum of } m \text{ and } n\\ 5\left(m+n\right)\end{array}$$

ⓑ To take a sum, we look for the words "of" and "and" to see what is being added. Here we are taking the sum *of* five times *m* and *n*.
Equation:

$$\begin{array}{c}\text{the sum of five times } m \text{ and } n\\ 5m+n\end{array}$$

Note:
Exercise:

Problem:

Translate the English phrase into an algebraic expression: ⓐ four times the sum of *p* and *q* ⓑ the sum of four times *p* and *q*.

Solution:

ⓐ $4\left(p+q\right)$ ⓑ $4p+q$

Note:
Exercise:

Problem:

Translate the English phrase into an algebraic expression: ⓐ the difference of two times *x* and 8, ⓑ two times the difference of *x* and 8.

Solution:

ⓐ $2x-8$ ⓑ $2\left(x-8\right)$

Later in this course, we'll apply our skills in algebra to solving applications. The first step will be to translate an English phrase to an algebraic expression. We'll see how to do this in the next two examples.

Example:
Exercise:

Problem:

The length of a rectangle is 6 less than the width. Let *w* represent the width of the rectangle. Write an expression for the length of the rectangle.

Solution:
Solution

Write a phrase about the length of the rectangle.	6 less than the width
Substitute w for "the width."	6 less than w
Rewrite "less than" as "subtracted from."	6 subtracted from w
Translate the phrase into algebra.	$w-6$

Note:
Exercise:

Problem:

The length of a rectangle is 7 less than the width. Let *w* represent the width of the rectangle. Write an expression for the length of the rectangle.

Solution:

$w-7$

Note:
Exercise:

Problem:

The width of a rectangle is 6 less than the length. Let *l* represent the length of the rectangle. Write an expression for the width of the rectangle.

Solution:

$l-6$

Example:
Exercise:

Problem:

June has dimes and quarters in her purse. The number of dimes is three less than four times the number of quarters. Let *q* represent the number of quarters. Write an expression for the number of dimes.

Solution:
Solution

Write the phrase about the number of dimes.	three less than four times the number of quarte
Substitute q for the number of quarters.	3 less than 4 times q
Translate “4 times q.”	3 less than $4q$
Translate the phrase into algebra.	$4q-3$

Note:
Exercise:

Problem:

Geoffrey has dimes and quarters in his pocket. The number of dimes is eight less than four times the number of quarters. Let q represent the number of quarters. Write an expression for the number of dimes.

Solution:

$4q-8$

Note:
Exercise:

Problem:

Lauren has dimes and nickels in her purse. The number of dimes is three more than seven times the number of nickels. Let n represent the number of nickels. Write an expression for the number of dimes.

Solution:

$7n+3$

Key Concepts

- **Notation** **The result is…**
  - $a+b$ the sum of a and b
  - $a-b$ the difference of a and b
  - $a\cdot b, ab, (a)\,(b)\;(a)b, a\,(b)$ the product of a and b
  - $a\div b, a/b, \frac{a}{b}, b\overline{)a}$ the quotient of a and b
- **Inequality**
  - $a<b$ is read “a is less than b” a is to the left of b on the number line
  - $a>b$ is read “a is greater than b” a is to the right of b on the number line
- **Inequality Symbols** **Words**

- $a \neq b$ — a is **not equal to** b
- $a < b$ — a is **less than** b
- $a \leq b$ — a is **less than or equal to** b
- $a > b$ — a is **greater than** b
- $a \geq b$ — a is **greater than or equal to** b

- **Grouping Symbols**
  - Parentheses ( )
  - Brackets [ ]
  - Braces { }
- **Exponential Notation**
  - a^n means multiply a by itself, n times. The expression a^n is read a to the n^{th} power.
- **Order of Operations:** When simplifying mathematical expressions perform the operations in the following order:

  Parentheses and other Grouping Symbols: Simplify all expressions inside the parentheses or other grouping symbols, working on the innermost parentheses first.
  Exponents: Simplify all expressions with exponents.
  Multiplication and Division: Perform all multiplication and division in order from left to right. These operations have equal priority.
  Addition and Subtraction: Perform all addition and subtraction in order from left to right. These operations have equal priority.
- **Combine Like Terms**

  Identify like terms.
  Rearrange the expression so like terms are together.
  Add or subtract the coefficients and keep the same variable for each group of like terms.

Practice Makes Perfect

Use Variables and Algebraic Symbols

In the following exercises, translate from algebra to English.
Exercise:

Problem: $16 - 9$

Solution:

16 minus 9, the difference of sixteen and nine

Exercise:

Problem: $3 \cdot 9$

Exercise:

Problem: $28 \div 4$

Solution:

28 divided by 4, the quotient of twenty-eight and four

Exercise:

Problem: $x + 11$

Exercise:

Problem: $(2)\,(7)$

Solution:

2 times 7, the product of two and seven

Exercise:

Problem: $(4)\,(8)$

Exercise:

Problem: $14 < 21$

Solution:

fourteen is less than twenty-one

Exercise:

Problem: $17 < 35$

Exercise:

Problem: $36 \geq 19$

Solution:

thirty-six is greater than or equal to nineteen

Exercise:

Problem: $6n = 36$

Exercise:

Problem: $y - 1 > 6$

Solution:

y minus 1 is greater than 6, the difference of *y* and one is greater than six

Exercise:

Problem: $y - 4 > 8$

Exercise:

Problem: $2 \leq 18 \div 6$

Solution:

2 is less than or equal to 18 divided by 6; 2 is less than or equal to the quotient of eighteen and six

Exercise:

Problem: $a \neq 1 \cdot 12$

In the following exercises, determine if each is an expression or an equation.

Exercise:

Problem: $9 \cdot 6 = 54$

Solution:

equation

Exercise:

Problem: $7 \cdot 9 = 63$

Exercise:

Problem: $5 \cdot 4 + 3$

Solution:

expression

Exercise:

Problem: $x + 7$

Exercise:

Problem: $x + 9$

Solution:

expression

Exercise:

Problem: $y - 5 = 25$

Simplify Expressions Using the Order of Operations

In the following exercises, simplify each expression.

Exercise:

Problem: 5^3

Solution:

125

Exercise:

Problem: 8^3

Exercise:

Problem: 2^8

Solution:

256

Exercise:

Problem: 10^5

In the following exercises, simplify using the order of operations.

Exercise:

Problem: ⓐ $3 + 8 \cdot 5$ ⓑ $(3 + 8) \cdot 5$

Solution:

ⓐ 43 ⓑ 55

Exercise:

Problem: ⓐ $2 + 6 \cdot 3$ ⓑ $(2 + 6) \cdot 3$

Exercise:

Problem: $2^3 - 12 \div (9 - 5)$

Solution:

5

Exercise:

Problem: $3^2 - 18 \div (11 - 5)$

Exercise:

Problem: $3 \cdot 8 + 5 \cdot 2$

Solution:

34

Exercise:

Problem: $4 \cdot 7 + 3 \cdot 5$

Exercise:

Problem: $2 + 8(6 + 1)$

Solution:

58

Exercise:

Problem: $4+6(3+6)$

Exercise:

Problem: $4 \cdot 12/8$

Solution:

6

Exercise:

Problem: $2 \cdot 36/6$

Exercise:

Problem: $(6+10) \div (2+2)$

Solution:

4

Exercise:

Problem: $(9+12) \div (3+4)$

Exercise:

Problem: $20 \div 4 + 6 \cdot 5$

Solution:

35

Exercise:

Problem: $33 \div 3 + 8 \cdot 2$

Exercise:

Problem: $3^2 + 7^2$

Solution:

58

Exercise:

Problem: $(3+7)^2$

Exercise:

Problem: $3(1+9 \cdot 6) - 4^2$

Solution:

149

Exercise:

Problem: $5(2+8\cdot 4)-7^2$

Exercise:

Problem: $2\left[1+3\left(10-2\right)\right]$

Solution:

50

Exercise:

Problem: $5\left[2+4\left(3-2\right)\right]$

Evaluate an Expression

In the following exercises, evaluate the following expressions.

Exercise:

Problem: $7x+8$ when $x=2$

Solution:

22

Exercise:

Problem: $8x-6$ when $x=7$

Exercise:

Problem: x^2 when $x=12$

Solution:

144

Exercise:

Problem: x^3 when $x=5$

Exercise:

Problem: x^5 when $x=2$

Solution:

32

Exercise:

Problem: 4^x when $x=2$

Exercise:

Problem: $x^2 + 3x - 7$ when $x = 4$

Solution:

21

Exercise:

Problem: $6x + 3y - 9$ when $x = 6, y = 9$

Exercise:

Problem: $(x - y)^2$ when $x = 10, y = 7$

Solution:

9

Exercise:

Problem: $(x + y)^2$ when $x = 6, y = 9$

Exercise:

Problem: $a^2 + b^2$ when $a = 3, b = 8$

Solution:

73

Exercise:

Problem: $r^2 - s^2$ when $r = 12, s = 5$

Exercise:

Problem: $2l + 2w$ when $l = 15, w = 12$

Solution:

54

Exercise:

Problem: $2l + 2w$ when $l = 18, w = 14$

Simplify Expressions by Combining Like Terms

In the following exercises, identify the coefficient of each term.

Exercise:

Problem: $8a$

Solution:

8

Exercise:

Problem: $13m$

Exercise:

Problem: $5r^2$

Solution:

5

Exercise:

Problem: $6x^3$

In the following exercises, identify the like terms.

Exercise:

Problem: $x^3, 8x, 14, 8y, 5, 8x^3$

Solution:

x^3 and $8x^3$, 14 and 5

Exercise:

Problem: $6z, 3w^2, 1, 6z^2, 4z, w^2$

Exercise:

Problem: $9a, a^2, 16, 16b^2, 4, 9b^2$

Solution:

16 and 4, $16b^2$ and $9b^2$

Exercise:

Problem: $3, 25r^2, 10s, 10r, 4r^2, 3s$

In the following exercises, identify the terms in each expression.

Exercise:

Problem: $15x^2 + 6x + 2$

Solution:

$15x^2, 6x, 2$

Exercise:

Problem: $11x^2 + 8x + 5$

Exercise:

Problem: $10y^3 + y + 2$

Solution:

$10y^3, y, 2$

Exercise:

Problem: $9y^3 + y + 5$

In the following exercises, simplify the following expressions by combining like terms.

Exercise:

Problem: $10x + 3x$

Solution:

$13x$

Exercise:

Problem: $15x + 4x$

Exercise:

Problem: $4c + 2c + c$

Solution:

$7c$

Exercise:

Problem: $6y + 4y + y$

Exercise:

Problem: $7u + 2 + 3u + 1$

Solution:

$10u + 3$

Exercise:

Problem: $8d + 6 + 2d + 5$

Exercise:

Problem: $10a + 7 + 5a - 2 + 7a - 4$

Solution:

$22a + 1$

Exercise:

Problem: $7c + 4 + 6c - 3 + 9c - 1$

Exercise:

Problem: $3x^2 + 12x + 11 + 14x^2 + 8x + 5$

Solution:

$17x^2 + 20x + 16$

Exercise:

Problem: $5b^2 + 9b + 10 + 2b^2 + 3b - 4$

Translate an English Phrase to an Algebraic Expression

In the following exercises, translate the phrases into algebraic expressions.

Exercise:

Problem: the difference of 14 and 9

Solution:

$14 - 9$

Exercise:

Problem: the difference of 19 and 8

Exercise:

Problem: the product of 9 and 7

Solution:

$9 \cdot 7$

Exercise:

Problem: the product of 8 and 7

Exercise:

Problem: the quotient of 36 and 9

Solution:

$36 \div 9$

Exercise:

Problem: the quotient of 42 and 7

Exercise:

Problem: the sum of *8x* and 3*x*

Solution:

$8x + 3x$

Exercise:

Problem: the sum of *13x* and 3*x*

Exercise:

Problem: the quotient of *y* and 3

Solution:

$\frac{y}{3}$

Exercise:

Problem: the quotient of *y* and 8

Exercise:

Problem: eight times the difference of *y* and nine

Solution:

$8\left(y-9\right)$

Exercise:

Problem: seven times the difference of *y* and one

Exercise:

Problem:

Eric has rock and classical CDs in his car. The number of rock CDs is 3 more than the number of classical CDs. Let *c* represent the number of classical CDs. Write an expression for the number of rock CDs.

Solution:

$b-4$

Exercise:

Problem:

The number of girls in a second-grade class is 4 less than the number of boys. Let *b* represent the number of boys. Write an expression for the number of girls.

Exercise:

Problem:

Greg has nickels and pennies in his pocket. The number of pennies is seven less than twice the number of nickels. Let n represent the number of nickels. Write an expression for the number of pennies.

Solution:

$2n - 7$

Exercise:

Problem:

Jeannette has $5 and $10 bills in her wallet. The number of fives is three more than six times the number of tens. Let *t* represent the number of tens. Write an expression for the number of fives.

Everyday Math

Exercise:

Problem:

Car insurance Justin's car insurance has a $750 deductible per incident. This means that he pays $750 and his insurance company will pay all costs beyond $750. If Justin files a claim for $2,100.

ⓐ how much will he pay?
ⓑ how much will his insurance company pay?

Solution:

ⓐ $750 ⓑ $1,350

Exercise:

Problem:

Home insurance Armando's home insurance has a $2,500 deductible per incident. This means that he pays $2,500 and the insurance company will pay all costs beyond $2,500. If Armando files a claim for $19,400.

ⓐ how much will he pay?
ⓑ how much will the insurance company pay?

Writing Exercises

Exercise:

Problem: Explain the difference between an expression and an equation.

Solution:

Answers may vary

Exercise:

Problem: Why is it important to use the order of operations to simplify an expression?

Exercise:

Problem: Explain how you identify the like terms in the expression $8a^2 + 4a + 9 - a^2 - 1$.

Solution:

Answers may vary

Exercise:

Problem:

Explain the difference between the phrases “4 times the sum of *x* and *y*” and “the sum of 4 times *x* and *y*.”

Self Check

ⓐ Use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
use variables and algebraic symbols.			
simplify expressions using the order of operations.			
evaluate an expression.			
identify and combine like terms.			
translate English phrases to algebraic expressions.			

ⓑ After reviewing this checklist, what will you do to become confident for all objectives?

Glossary

coefficient
: The coefficient of a term is the constant that multiplies the variable in a term.

constant
: A constant is a number whose value always stays the same.

equality symbol
: The symbol “$=$” is called the equal sign. We read $a=b$ as “a is equal to b.”

equation
: An equation is two expressions connected by an equal sign.

evaluate an expression
: To evaluate an expression means to find the value of the expression when the variable is replaced by a given number.

expression
: An expression is a number, a variable, or a combination of numbers and variables using operation symbols.

like terms
: Terms that are either constants or have the same variables raised to the same powers are called like terms.

simplify an expression
: To simplify an expression, do all operations in the expression.

term
: A term is a constant or the product of a constant and one or more variables.

variable
: A variable is a letter that represents a number whose value may change.

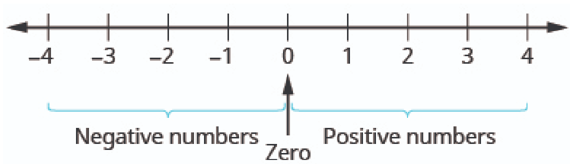
Add and Subtract Integers
By the end of this section, you will be able to:

- Use negatives and opposites
- Simplify: expressions with absolute value
- Add integers
- Subtract integers

Note:
A more thorough introduction to the topics covered in this section can be found in the *Prealgebra* chapter, **Integers**.

Use Negatives and Opposites

Our work so far has only included the counting numbers and the whole numbers. But if you have ever experienced a temperature below zero or accidentally overdrawn your checking account, you are already familiar with negative numbers. **Negative numbers** are numbers less than 0. The negative numbers are to the left of zero on the number line. See [link].

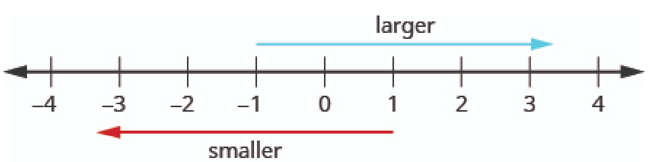


The number line shows the location of positive and negative numbers.

The arrows on the ends of the number line indicate that the numbers keep going forever. There is no biggest positive number, and there is no smallest negative number.

Is zero a positive or a negative number? Numbers larger than zero are positive, and numbers smaller than zero are negative. Zero is neither positive nor negative.

Consider how numbers are ordered on the number line. Going from left to right, the numbers increase in value. Going from right to left, the numbers decrease in value. See [link].



The numbers on a number line increase in value going from left to right and decrease in value going from right to left.

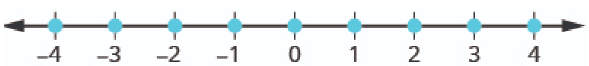
Note: Doing the Manipulative Mathematics activity “Number Line-part 2” will help you develop a better understanding of integers.

Remember that we use the notation:

$a < b$ (read “a is less than b”) when a is to the left of b on the number line.

$a > b$ (read “*a* is greater than *b*”) when *a* is to the right of *b* on the number line.

Now we need to extend the number line which showed the whole numbers to include negative numbers, too. The numbers marked by points in [link] are called the integers. The integers are the numbers $\ldots -3, -2, -1, 0, 1, 2, 3\ldots$



All the marked numbers are called *integers*.

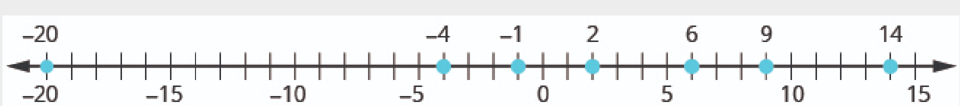
Example:
Exercise:

Problem:

Order each of the following pairs of numbers, using < or >: ⓐ 14 ___ 6 ⓑ -1 ___ 9 ⓒ -1 ___ -4 ⓓ 2 ___ -20.

Solution:
Solution

It may be helpful to refer to the number line shown.



ⓐ

$$14 \_\_\_ 6$$

14 is to the right of 6 on the number line. $14 > 6$

ⓑ

$$-1 \_\_\_ 9$$

-1 is to the left of 9 on the number line. $-1 < 9$

ⓒ

$$-1 \_\_\_ -4$$

-1 is to the right of -4 on the number line. $-1 > -4$

ⓓ

$$2 \_\_\_ -20$$

2 is to the right of -20 on the number line. $2 > -20$

Note:
Exercise:

Problem:

Order each of the following pairs of numbers, using $<$ or $>$: ⓐ $15 \_\_\_ 7$ ⓑ $-2 \_\_\_ 5$ ⓒ $-3 \_\_\_ -7$
ⓓ $5 \_\_\_ -17$.

Solution:

ⓐ > ⓑ < ⓒ > ⓓ >

Note:
Exercise:

Problem:

Order each of the following pairs of numbers, using $<$ or $>$: ⓐ $8 \_\_\_ 13$ ⓑ $3 \_\_\_ -4$ ⓒ $-5 \_\_\_ -2$
ⓓ $9 \_\_\_ -21$.

Solution:

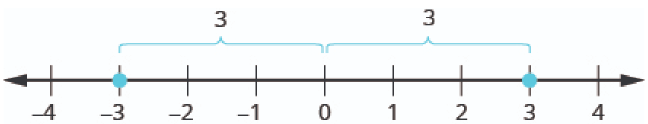
ⓐ < ⓑ > ⓒ < ⓓ >

You may have noticed that, on the number line, the negative numbers are a mirror image of the positive numbers, with zero in the middle. Because the numbers 2 and -2 are the same distance from zero, they

are called **opposites**. The opposite of 2 is -2, and the opposite of -2 is 2.

Note:
Opposite
The **opposite** of a number is the number that is the same distance from zero on the number line but on the opposite side of zero.

[link] illustrates the definition.



The opposite of 3 is -3.

Sometimes in algebra the same symbol has different meanings. Just like some words in English, the specific meaning becomes clear by looking at how it is used. You have seen the symbol "–" used in three different ways.
Equation:

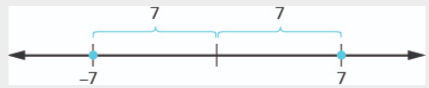
$$\begin{array}{ll} 10-4 & \text{Between two numbers, it indicates the operation of } \textit{subtraction}. \\ & \text{We read } 10-4 \text{ as "10 minus 4."} \\ -8 & \text{In front of a number, it indicates a } \textit{negative} \text{ number.} \\ & \text{We read } -8 \text{ as "negative eight."} \\ -x & \text{In front of a variable, it indicates the } \textit{opposite}. \text{ We read } -x \text{ as "the opposite of } x." \\ -(-2) & \text{Here there are two " } - \text{ " signs. The one in the parentheses tells us the number is} \\ & \text{negative } 2. \text{ The one outside the parentheses tells us to take the } \textit{opposite} \text{ of } -2. \\ & \text{We read } -(-2) \text{ as "the opposite of negative two."} \end{array}$$

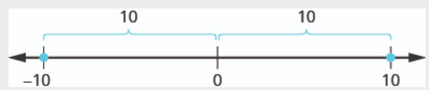
Note:
Opposite Notation
$-a$ means the opposite of the number a.
The notation $-a$ is read as "the opposite of a."

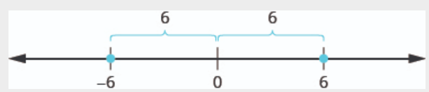
Example:
Exercise:

Problem: Find: ⓐ the opposite of 7 ⓑ the opposite of -10 ⓒ $-(-6)$.

Solution:
Solution

<table>
<tr>
<td>ⓐ −7 is the same distance from 0 as 7, but on the opposite side of 0.</td>
<td>

The opposite of 7 is −7.</td>
</tr>
<tr>
<td>ⓑ 10 is the same distance from 0 as −10, but on the opposite side of 0.</td>
<td>

The opposite of −10 is 10.</td>
</tr>
<tr>
<td>ⓒ −(−6)</td>
<td>

The opposite of −(−6) is −6.</td>
</tr>
</table>

Note:
Exercise:

Problem: Find: ⓐ the opposite of 4 ⓑ the opposite of -3 ⓒ $-(-1)$.

Solution:

ⓐ -4 ⓑ 3 ⓒ 1

Note:
Exercise:

Problem: Find: ⓐ the opposite of 8 ⓑ the opposite of -5 ⓒ $-(-5)$.

Solution:

ⓐ -8 ⓑ 5 ⓒ 5

Our work with opposites gives us a way to define the integers.The whole numbers and their opposites are called the **integers**. The integers are the numbers $\ldots -3, -2, -1, 0, 1, 2, 3 \ldots$

Note:
Integers
The whole numbers and their opposites are called the **integers**.
The integers are the numbers
Equation:

$$\ldots -3, -2, -1, 0, 1, 2, 3 \ldots$$

When evaluating the opposite of a variable, we must be very careful. Without knowing whether the variable represents a positive or negative number, we don't know whether $-x$ is positive or negative. We can see this in [link].

Example:
Exercise:

Problem: Evaluate ⓐ $-x$, when $x = 8$ ⓑ $-x$, when $x = -8$.

Solution:
Solution

ⓐ

To evaluate when $x = 8$ means to substitute 8 for x.	
	$-x$

Substitute 8 for *x*.	–(8)
Write the opposite of 8.	–8

ⓑ

To evaluate when *x* = –8 means to substitute –8 for –*x*.	
	$-x$
Substitute –8 for *x*.	–(–8)
Write the opposite of −8.	8

Note:
Exercise:

Problem: Evaluate $-n$, when ⓐ $n = 4$ ⓑ $n = -4$.

Solution:

ⓐ -4 ⓑ 4

Note:
Exercise:

Problem: Evaluate $-m$, when ⓐ $m = 11$ ⓑ $m = -11$.

Solution:

ⓐ -11 ⓑ 11

Simplify: Expressions with Absolute Value

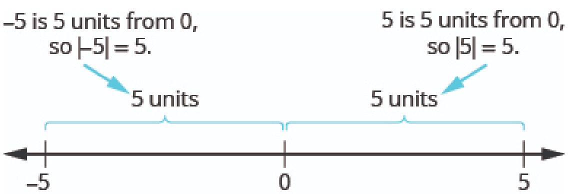
We saw that numbers such as 2 and -2 are opposites because they are the same distance from 0 on the number line. They are both two units from 0. The distance between 0 and any number on the number line is called the **absolute value** of that number.

Note:
Absolute Value
The **absolute value** of a number is its distance from 0 on the number line.
The absolute value of a number *n* is written as $|n|$.

For example,

- -5 is 5 units away from 0, so $|-5| = 5$.
- 5 is 5 units away from 0, so $|5| = 5$.

[link] illustrates this idea.



The integers 5 and are 5 units away from 0.

The absolute value of a number is never negative (because distance cannot be negative). The only number with absolute value equal to zero is the number zero itself, because the distance from 0 to 0 on the number line is zero units.

Note:
Property of Absolute Value
$|n| \geq 0$ for all numbers
Absolute values are always greater than or equal to zero!

Mathematicians say it more precisely, “absolute values are always non-negative.” Non-negative means greater than or equal to zero.

Example:
Exercise:

Problem: Simplify: ⓐ $|3|$ ⓑ $|-44|$ ⓒ $|0|$.

Solution:
Solution

The absolute value of a number is the distance between the number and zero. Distance is never negative, so the absolute value is never negative.

ⓐ $|3|$
3

ⓑ $|-44|$
44

ⓒ $|0|$
0

Note:
Exercise:

Problem: Simplify: ⓐ $|4|$ ⓑ $|-28|$ ⓒ $|0|$.

Solution:

ⓐ 4 ⓑ 28 ⓒ 0

Note:
Exercise:

Problem: Simplify: ⓐ $|-13|$ ⓑ $|47|$ ⓒ $|0|$.

Solution:

ⓐ 13 ⓑ 47 ⓒ 0

In the next example, we’ll order expressions with absolute values. Remember, positive numbers are always greater than negative numbers!

Example:

Exercise:

Problem: Fill in $<, >,$ or $=$ for each of the following pairs of numbers:

ⓐ $|-5|$ ___ $-|-5|$ ⓑ 8 ___ $-|-8|$ ⓒ -9 ___ $-|-9|$ ⓓ $-(-16)$ ___ $-|-16|$

Solution:
Solution

ⓐ

$$\begin{array}{lrcl} & |-5| & \_\_\_ & -|-5| \\ \text{Simplify.} & 5 & \_\_\_ & -5 \\ \text{Order.} & 5 & > & -5 \\ & |-5| & > & -|-5| \end{array}$$

ⓑ

$$\begin{array}{lrcl} & 8 & \_\_\_ & -|-8| \\ \text{Simplify.} & 8 & \_\_\_ & -8 \\ \text{Order.} & 8 & > & -8 \\ & 8 & > & -|-8| \end{array}$$

ⓒ

$$\begin{array}{lrcl} & 9 & \_\_\_ & -|-9| \\ \text{Simplify.} & -9 & \_\_\_ & -9 \\ \text{Order.} & -9 & = & -9 \\ & -9 & = & -|-9| \end{array}$$

ⓓ

$$\begin{array}{lrcl} & -(-16) & \_\_\_ & -|-16| \\ \text{Simplify.} & 16 & \_\_\_ & -16 \\ \text{Order.} & 16 & > & -16 \\ & -(-16) & > & -|-16| \end{array}$$

Note:
Exercise:

Problem:

Fill in <, >, or = for each of the following pairs of numbers: ⓐ $|-9|$ ___ $-|-9|$ ⓑ 2 ___ $-|-2|$
ⓒ -8 ___ $|-8|$
ⓓ $-(-9)$ ___ $-|-9|$.

Solution:

ⓐ > ⓑ > ⓒ < ⓓ >

Note:
Exercise:

Problem:

Fill in <, >, or = for each of the following pairs of numbers: ⓐ 7 ___ $-|-7|$ ⓑ $-(-10)$ ___ $-|-10|$
ⓒ $|-4|$ ___ $-|-4|$ ⓓ -1 ___ $|-1|$.

Solution:

ⓐ > ⓑ > ⓒ > ⓓ <

We now add absolute value bars to our list of grouping symbols. When we use the order of operations, first we simplify inside the absolute value bars as much as possible, then we take the absolute value of the resulting number.

Note:
Grouping Symbols
Equation:

$$\begin{array}{llll} \text{Parentheses} & () & \text{Braces} & \{\} \\ \text{Brackets} & [] & \text{Absolute value} & |\,| \end{array}$$

In the next example, we simplify the expressions inside absolute value bars first, just like we do with parentheses.

Example:
Exercise:

Problem: Simplify: $24 - |19 - 3(6 - 2)|$.

Solution:
Solution

$$\begin{array}{lc} & 24 - |19 - 3(6 - 2)| \\ \text{Work inside parentheses first: subtract 2 from 6.} & 24 - |19 - 3(4)| \\ \text{Multiply } 3(4). & 24 - |19 - 12| \\ \text{Subtract inside the absolute value bars.} & 24 - |7| \\ \text{Take the absolute value.} & 24 - 7 \\ \text{Subtract.} & 17 \end{array}$$

Note:
Exercise:

Problem: Simplify: $19 - |11 - 4(3 - 1)|$.

Solution:

16

Note:
Exercise:

Problem: Simplify: $9 - |8 - 4(7 - 5)|$.

Solution:

9

Example:
Exercise:

Problem:

Evaluate: ⓐ $|x|$ when $x = -35$ ⓑ $|-y|$ when $y = -20$ ⓒ $-|u|$ when $u = 12$ ⓓ $-|p|$ when $p = -14$.

Solution:
Solution

ⓐ $|x|$ when $x = -35$

	$\lvert x\rvert$
Substitute −35 for *x*.	$\lvert -35\rvert$
Take the absolute value.	35

ⓑ $|-y|$ when $y = -20$

	$|-y|$
Substitute -20 for y.	$\left|-(-20)\right|$
Simplify.	$|20|$
Take the absolute value.	20

ⓒ $-|u|$ when $u = 12$

	$-|u|$
Substitute 12 for u.	$-\left|12\right|$
Take the absolute value.	-12

ⓓ $-|p|$ when $p = -14$

	$-|p|$
Substitute -14 for p.	$-|-14|$
Take the absolute value.	-14

Note:
Exercise:

Problem:

Evaluate: ⓐ $|x|$ when $x=-17$ ⓑ $|-y|$ when $y=-39$ ⓒ $-|m|$ when $m=22$ ⓓ $-|p|$ when $p=-11$.

Solution:

ⓐ 17 ⓑ 39 ⓒ -22 ⓓ -11

Note:
Exercise:

Problem:

Evaluate: ⓐ $|y|$ when $y=-23$ ⓑ $|-y|$ when $y=-21$ ⓒ $-|n|$ when $n=37$ ⓓ $-|q|$ when $q=-49$.

Solution:

ⓐ 23 ⓑ 21 ⓒ -37 ⓓ -49

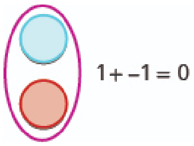
Add Integers

Most students are comfortable with the addition and subtraction facts for positive numbers. But doing addition or subtraction with both positive and negative numbers may be more challenging.

Note: Doing the Manipulative Mathematics activity “Addition of Signed Numbers” will help you develop a better understanding of adding integers.”

We will use two color counters to model addition and subtraction of negatives so that you can visualize the procedures instead of memorizing the rules.

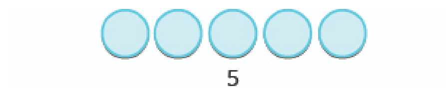
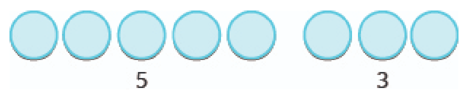
We let one color (blue) represent positive. The other color (red) will represent the negatives. If we have one positive counter and one negative counter, the value of the pair is zero. They form a neutral pair. The value of this neutral pair is zero.



We will use the counters to show how to add the four addition facts using the numbers $5, -5$ and $3, -3$.
Equation:

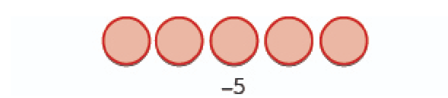
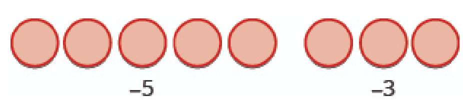
$$5+3 \qquad -5+(-3) \qquad -5+3 \qquad 5+(-3)$$

To add $5+3$, we realize that $5+3$ means the sum of 5 and 3.

We start with 5 positives.	
And then we add 3 positives.	
We now have 8 positives. The sum of 5 and 3 is 8.	

Now we will add $-5+(-3)$. Watch for similarities to the last example $5+3=8$.

To add $-5+(-3)$, we realize this means the sum of -5 and -3.

We start with 5 negatives.	
And then we add 3 negatives.	

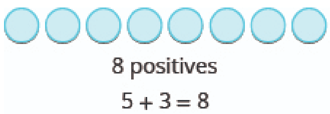
We now have 8 negatives. The sum of −5 and −3 is −8.	

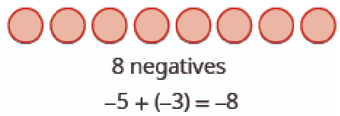
In what ways were these first two examples similar?

- The first example adds 5 positives and 3 positives—both positives.
- The second example adds 5 negatives and 3 negatives—both negatives.

In each case we got 8—either 8 positives or 8 negatives.

When the signs were the same, the counters were all the same color, and so we added them.



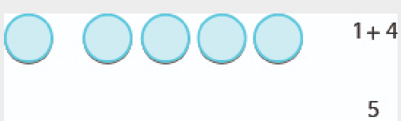


Example:
Exercise:

Problem: Add: ⓐ $1+4$ ⓑ $-1+(-4)$.

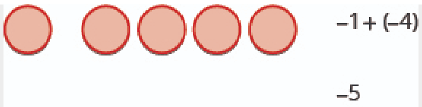
Solution:
Solution

ⓐ



1 positive plus 4 positives is 5 positives.

ⓑ



1 negative plus 4 negatives is 5 negatives.

Note:
Exercise:

Problem: Add: ⓐ $2+4$ ⓑ $-2+(-4)$.

Solution:

ⓐ 6 ⓑ -6

Note:
Exercise:

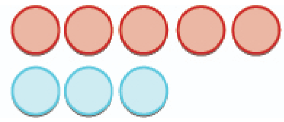
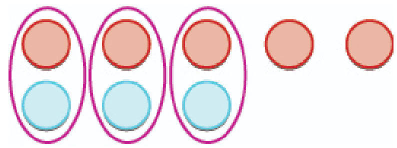
Problem: Add: ⓐ $2+5$ ⓑ $-2+(-5)$.

Solution:

ⓐ 7 ⓑ -7

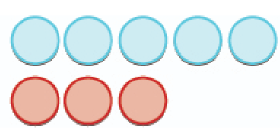
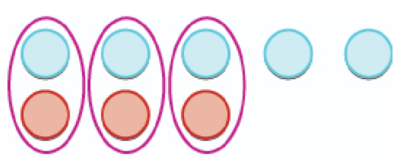
So what happens when the signs are different? Let’s add $-5+3$. We realize this means the sum of -5 and 3. When the counters were the same color, we put them in a row. When the counters are a different color, we line them up under each other.

	−5 + 3 means the sum of −5 and 3.
We start with 5 negatives.	
And then we add 3 positives.	

	
We remove any neutral pairs.	
We have 2 negatives left.	
The sum of −5 and 3 is −2.	−5 + 3 = −2

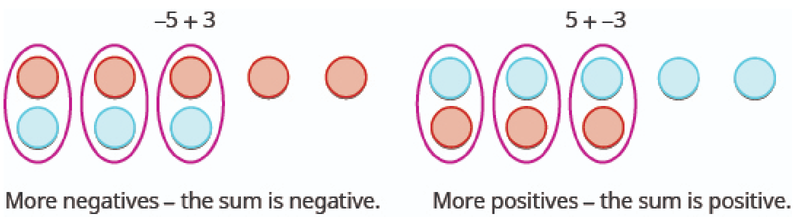
Notice that there were more negatives than positives, so the result was negative.

Let’s now add the last combination, $5 + (-3)$.

	5 + (−3) means the sum of 5 and −3.
We start with 5 positives.	
And then we add 3 negatives.	
We remove any neutral pairs.	
We have 2 positives left.	

The sum of 5 and −3 is 2.	5 + (−3) = 2

When we use counters to model addition of positive and negative integers, it is easy to see whether there are more positive or more negative counters. So we know whether the sum will be positive or negative.

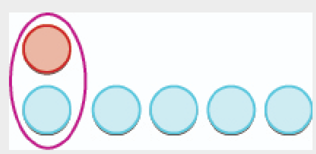


Example:
Exercise:

Problem: Add: ⓐ $-1+5$ ⓑ $1+(-5)$.

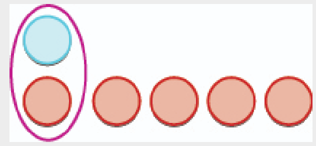
Solution:
Solution

ⓐ

	−1 + 5
	
There are more positives, so the sum is positive.	4

ⓑ

	1 + (−5)

	
There are more negatives, so the sum is negative.	−4

Note:
Exercise:

Problem: Add: ⓐ $-2+4$ ⓑ $2+(-4)$.

Solution:

ⓐ 2 ⓑ -2

Note:
Exercise:

Problem: Add: ⓐ $-2+5$ ⓑ $2+(-5)$.

Solution:

ⓐ 3 ⓑ -3

Now that we have added small positive and negative integers with a model, we can visualize the model in our minds to simplify problems with any numbers.

When you need to add numbers such as $37+(-53)$, you really don't want to have to count out 37 blue counters and 53 red counters. With the model in your mind, can you visualize what you would do to solve the problem?

Picture 37 blue counters with 53 red counters lined up underneath. Since there would be more red (negative) counters than blue (positive) counters, the sum would be *negative*. How many more red counters would there be? Because $53-37=16$, there are 16 more red counters.

Therefore, the sum of $37+(-53)$ is -16.
Equation:

$$37+(-53)=-16$$

Let's try another one. We'll add $-74+(-27)$. Again, imagine 74 red counters and 27 more red counters, so we'd have 101 red counters. This means the sum is -101.
Equation:

$$-74+(-27)=-101$$

Let's look again at the results of adding the different combinations of $5, -5$ and $3, -3$.

Note:
Addition of Positive and Negative Integers
Equation:

$$\begin{array}{cc} 5+3 & -5+(-3) \\ 8 & -8 \\ \text{both positive, sum positive} & \text{both negative, sum negative} \end{array}$$

When the signs are the same, the counters would be all the same color, so add them.
Equation:

$$\begin{array}{cc} -5+3 & 5+(-3) \\ -2 & 2 \\ \text{different signs, more negatives, sum negative} & \text{different signs, more positives, sum positive} \end{array}$$

When the signs are different, some of the counters would make neutral pairs, so subtract to see how many are left.

Visualize the model as you simplify the expressions in the following examples.

Example:
Exercise:

Problem: Simplify: ⓐ $19+(-47)$ ⓑ $-14+(-36)$.

Solution:

ⓐ Since the signs are different, we subtract 19 from 47. The answer will be negative because there are more negatives than positives.

$$\begin{array}{ll} & 19+(-47) \\ \text{Add.} & -28 \end{array}$$

ⓑ Since the signs are the same, we add. The answer will be negative because there are only negatives.

$$\begin{array}{ll} & -14+(-36) \\ \text{Add.} & -50 \end{array}$$

Note:
Exercise:

Problem: Simplify: ⓐ $-31+(-19)$ ⓑ $15+(-32)$.

Solution:

ⓐ -50 ⓑ -17

Note:
Exercise:

Problem: Simplify: ⓐ $-42+(-28)$ ⓑ $25+(-61)$.

Solution:

ⓐ -70 ⓑ -36

The techniques used up to now extend to more complicated problems, like the ones we've seen before. Remember to follow the order of operations!

Example:
Exercise:

Problem: Simplify: $-5+3(-2+7)$.

Solution:
Solution

	$-5+3(-2+7)$
Simplify inside the parentheses.	$-5+3(5)$
Multiply.	$-5+15$
Add left to right.	10

Note:
Exercise:

Problem: Simplify: $-2+5\left(-4+7\right)$.

Solution:

13

Note:
Exercise:

Problem: Simplify: $-4+2\left(-3+5\right)$.

Solution:

0

Subtract Integers

Note: Doing the Manipulative Mathematics activity “Subtraction of Signed Numbers” will help you develop a better understanding of subtracting integers.

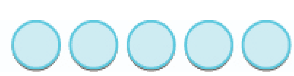
We will continue to use counters to model the subtraction. Remember, the blue counters represent positive numbers and the red counters represent negative numbers.

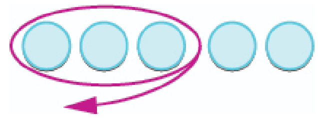
Perhaps when you were younger, you read “$5-3$” as “5 take away 3.” When you use counters, you can think of subtraction the same way!

We will model the four subtraction facts using the numbers 5 and 3.
Equation:

$$5-3 \qquad -5-\left(-3\right) \qquad -5-3 \qquad 5-\left(-3\right)$$

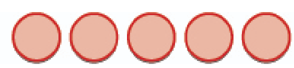
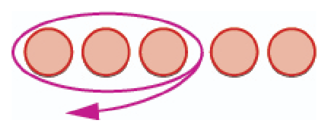
To subtract $5-3$, we restate the problem as “5 take away 3.”

We start with 5 positives.	

We ‘take away’ 3 positives.	
We have 2 positives left.	
The difference of 5 and 3 is 2.	2

Now we will subtract $-5-(-3)$. Watch for similarities to the last example $5-3=2$.

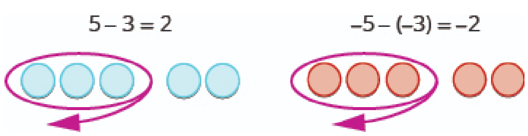
To subtract $-5-(-3)$, we restate this as “–5 take away –3”

We start with 5 negatives.	
We ‘take away’ 3 negatives.	
We have 2 negatives left.	
The difference of −5 and −3 is −2.	−2

Notice that these two examples are much alike: The first example, we subtract 3 positives from 5 positives and end up with 2 positives.

In the second example, we subtract 3 negatives from 5 negatives and end up with 2 negatives.

Each example used counters of only one color, and the “take away” model of subtraction was easy to apply.



Example:
Exercise:

Problem: Subtract: ⓐ $7 - 5$ ⓑ $-7 - (-5)$.

Solution:

ⓐ

	$7 - 5$
Take 5 positives from 7 positives and get 2 positives.	2

ⓑ

	$-7 - (-5)$
Take 5 negatives from 7 negatives and get 2 negatives.	-2

Note:
Exercise:

Problem: Subtract: ⓐ $6 - 4$ ⓑ $-6 - (-4)$.

Solution:

ⓐ 2 ⓑ -2

Note:
Exercise:

Problem: Subtract: ⓐ $7 - 4$ ⓑ $-7 - (-4)$.

Solution:

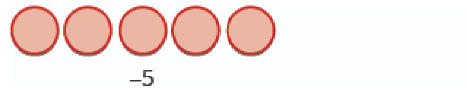
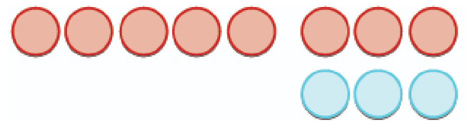
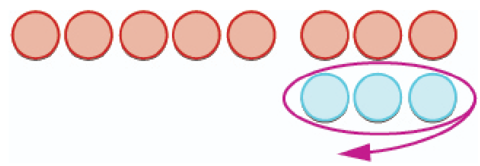
ⓐ 3 ⓑ -3

What happens when we have to subtract one positive and one negative number? We'll need to use both white and red counters as well as some neutral pairs. Adding a neutral pair does not change the value. It is like changing quarters to nickels—the value is the same, but it looks different.

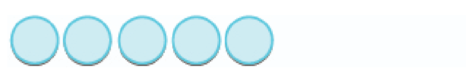
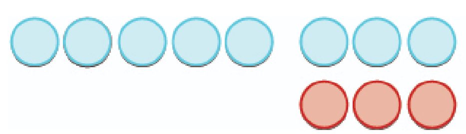
- To subtract $-5 - 3$, we restate it as -5 take away 3.

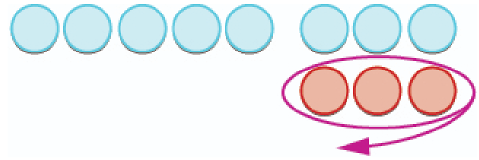
We start with 5 negatives. We need to take away 3 positives, but we do not have any positives to take away.

Remember, a neutral pair has value zero. If we add 0 to 5 its value is still 5. We add neutral pairs to the 5 negatives until we get 3 positives to take away.

	−5 − 3 means −5 take away 3.
We start with 5 negatives.	
We now add the neutrals needed to get 3 positives.	
We remove the 3 positives.	
We are left with 8 negatives.	
The difference of −5 and 3 is −8.	−5 − 3 = −8

And now, the fourth case, $5 - (-3)$. We start with 5 positives. We need to take away 3 negatives, but there are no negatives to take away. So we add neutral pairs until we have 3 negatives to take away.

	5 − (−3) means 5 take away −3.
We start with 5 positives.	
We now add the needed neutrals pairs.	
We remove the 3 negatives.	

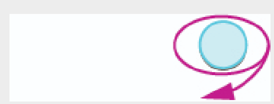
	
We are left with 8 positives.	
The difference of 5 and −3 is 8.	$5 - (-3) = 8$

Example:
Exercise:

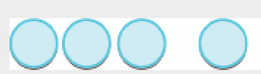
Problem: Subtract: ⓐ $-3 - 1$ ⓑ $3 - (-1)$.

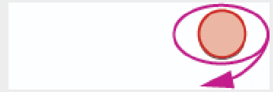
Solution:
Solution

ⓐ

Take 1 positive from the one added neutral pair.	 	$-3 - 1$ -4

ⓑ

Take 1 negative from the one added neutral pair.		$3 - (-1)$ 4



Note:
Exercise:

Problem: Subtract: ⓐ $-6-4$ ⓑ $6-(-4)$.

Solution:

ⓐ -10 ⓑ 10

Note:
Exercise:

Problem: Subtract: ⓐ $-7-4$ ⓑ $7-(-4)$.

Solution:

ⓐ -11 ⓑ 11

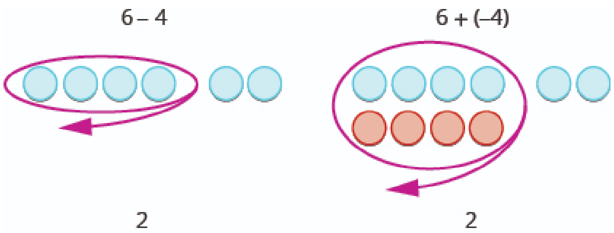
Have you noticed that *subtraction of signed numbers can be done by adding the opposite*? In [link], $-3-1$ is the same as $-3+(-1)$ and $3-(-1)$ is the same as $3+1$. You will often see this idea, the **subtraction property**, written as follows:

Note:
Subtraction Property
Equation:

$$a-b=a+(-b)$$

Subtracting a number is the same as adding its opposite.

Look at these two examples.



Equation:

$$6-4 \text{ gives the same answer as } 6+(-4).$$

Of course, when you have a subtraction problem that has only positive numbers, like $6-4$, you just do the subtraction. You already knew how to subtract $6-4$ long ago. But *knowing* that $6-4$ gives the same answer as $6+(-4)$ helps when you are subtracting negative numbers. Make sure that you understand how $6-4$ and $6+(-4)$ give the same results!

Example:
Exercise:

Problem: Simplify: ⓐ $13-8$ and $13+(-8)$ ⓑ $-17-9$ and $-17+(-9)$.

Solution:
Solution

ⓐ

	$13-8$	and	$13+(-8)$
Subtract.	5		5

ⓑ

	$-17-9$	and	$-17+(-9)$
Subtract.	-26		-26

Note:
Exercise:

Problem: Simplify: ⓐ $21-13$ and $21+(-13)$ ⓑ $-11-7$ and $-11+(-7)$.

Solution:

ⓐ 8 ⓑ −18

Note:

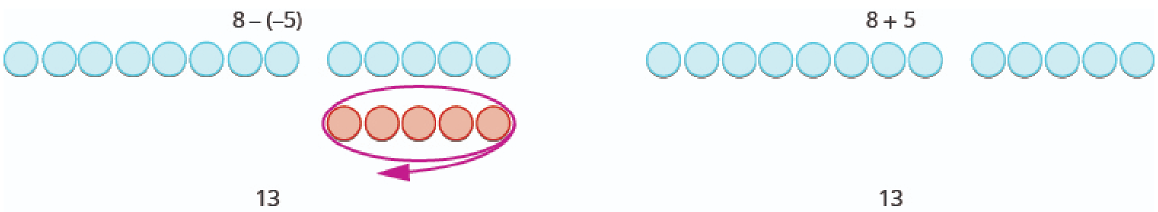
Exercise:

Problem: Simplify: ⓐ $15-7$ and $15+(-7)$ ⓑ $-14-8$ and $-14+(-8)$.

Solution:

ⓐ 8 ⓑ -22

Look at what happens when we subtract a negative.



Equation:

$$8-(-5) \text{ gives the same answer as } 8+5$$

Subtracting a negative number is like adding a positive!

You will often see this written as $a-(-b)=a+b$.

Does that work for other numbers, too? Let’s do the following example and see.

Example:
Exercise:

Problem: Simplify: ⓐ $9-(-15)$ and $9+15$ ⓑ $-7-(-4)$ and $-7+4$.

Solution:
Solution

ⓐ

	$9-(-15)$	$9+15$
Subtract.	24	24

ⓑ

	$-7-(-4)$	$-7+4$
Subtract.	-3	-3

Note:

Exercise:

Problem: Simplify: ⓐ $6-(-13)$ and $6+13$ ⓑ $-5-(-1)$ and $-5+1$.

Solution:

ⓐ 19 ⓑ -4

Note:
Exercise:

Problem: Simplify: ⓐ $4-(-19)$ and $4+19$ ⓑ $-4-(-7)$ and $-4+7$.

Solution:

ⓐ 23 ⓑ 3

Let's look again at the results of subtracting the different combinations of $5, -5$ and $3, -3$.

Note:
Subtraction of Integers
Equation:

$$\begin{array}{cc} 5-3 & -5-(-3) \\ 2 & -2 \\ \text{5 positives take away 3 positives} & \text{5 negatives take away 3 negatives} \\ \text{2 positives} & \text{2 negatives} \end{array}$$

When there would be enough counters of the color to take away, subtract.
Equation:

$$\begin{array}{cc} -5-3 & 5-(-3) \\ -8 & 8 \\ \text{5 negatives, want to take away 3 positives} & \text{5 positives, want to take away 3 negatives} \\ \text{need neutral pairs} & \text{need neutral pairs} \end{array}$$

When there would be not enough counters of the color to take away, add.

What happens when there are more than three integers? We just use the order of operations as usual.

Example:
Exercise:

Problem: Simplify: $7-(-4-3)-9$.

Solution:
Solution

	$7-(-4-3)-9$
Simplify inside the parentheses first.	$7-(-7)-9$
Subtract left to right.	$14-9$
Subtract.	5

Note:
Exercise:

Problem: Simplify: $8-(-3-1)-9$.

Solution:

3

Note:
Exercise:

Problem: Simplify: $12-(-9-6)-14$.

Solution:

13

Note:
Access these online resources for additional instruction and practice with adding and subtracting integers. You will need to enable Java in your web browser to use the applications.

- Add Colored Chip
- Subtract Colored Chip

Key Concepts

- **Addition of Positive and Negative Integers**

$5+3$	$-5+(-3)$
8	-8
both positive, sum positive	both negative, sum negative
$-5+3$	$5+(-3)$
-2	2
different signs, more negatives sum negative	different signs, more positives sum positive

- **Property of Absolute Value**: $|n| \geq 0$ for all numbers. Absolute values are always greater than or equal to zero!
- **Subtraction of Integers**

$5-3$	$-5-(-3)$
2	-2
5 positives take away 3 positives 2 positives	5 negatives take away 3 negatives 2 negatives
$-5-3$	$5-(-3)$
-8	8
5 negatives, want to subtract 3 positives need neutral pairs	5 positives, want to subtract 3 negatives need neutral pairs

- **Subtraction Property:** Subtracting a number is the same as adding its opposite.

Practice Makes Perfect

Use Negatives and Opposites of Integers

In the following exercises, order each of the following pairs of numbers, using < or >.

Exercise:

Problem:
ⓐ 9 ___ 4
ⓑ -3 ___ 6
ⓒ -8 ___ -2
ⓓ 1 ___ -10

Solution:

ⓐ > ⓑ < ⓒ < ⓓ >

Exercise:

ⓐ -7 ___ 3
ⓑ -10 ___ -5
ⓒ 2 ___ -6
Problem: ⓓ 8 ___ 9

In the following exercises, find the opposite of each number.

Exercise:

ⓐ 2
Problem: ⓑ -6

Solution:

ⓐ -2 ⓑ 6

Exercise:

ⓐ 9
Problem: ⓑ -4

In the following exercises, simplify.

Exercise:

Problem: $-(-4)$

Solution:

4

Exercise:

Problem: $-(-8)$

Exercise:

Problem: $-(-15)$

Solution:

15

Exercise:

Problem: $-(-11)$

In the following exercises, evaluate.

Exercise:

Problem: $-c$ when
ⓐ $c = 12$
ⓑ $c = -12$

Solution:

ⓐ -12 ⓑ 12

Exercise:

Problem: $-d$ when
ⓐ $d = 21$
ⓑ $d = -21$

Simplify Expressions with Absolute Value

In the following exercises, simplify.

Exercise:

Problem: ⓐ $|-32|$
ⓑ $|0|$
ⓒ $|16|$

Solution:

ⓐ 32 ⓑ 0 ⓒ 16

Exercise:

Problem: ⓐ $|0|$
ⓑ $|-40|$
ⓒ $|22|$

In the following exercises, fill in $<$, $>$, or $=$ for each of the following pairs of numbers.

Exercise:

Problem: ⓐ -6 ___ $|-6|$
ⓑ $-|-3|$ ___ -3

Solution:

ⓐ $<$ ⓑ $=$

Exercise:

Problem: ⓐ $|-5|$ ___ $-|-5|$
ⓑ 9 ___ $-|-9|$

In the following exercises, simplify.

Exercise:

Problem: $-(-5)$ and $-|-5|$

Solution:

$5, -5$

Exercise:

Problem: $-|-9|$ and $-(-9)$

Exercise:

Problem: $8|-7|$

Solution:

56

Exercise:

Problem: $5|-5|$

Exercise:

Problem: $|15-7|-|14-6|$

Solution:

0

Exercise:

Problem: $|17-8|-|13-4|$

Exercise:

Problem: $18-|2(8-3)|$

Solution:

8

Exercise:

Problem: $18 - |3(8 - 5)|$

In the following exercises, evaluate.

Exercise:

Problem: ⓐ $-|p|$ when $p = 19$
ⓑ $-|q|$ when $q = -33$

Solution:

ⓐ -19 ⓑ -33

Exercise:

Problem: ⓐ $-|a|$ when $a = 60$
ⓑ $-|b|$ when $b = -12$

Add Integers

In the following exercises, simplify each expression.

Exercise:

Problem: $-21 + (-59)$

Solution:

-80

Exercise:

Problem: $-35 + (-47)$

Exercise:

Problem: $48 + (-16)$

Solution:

32

Exercise:

Problem: $34 + (-19)$

Exercise:

Problem: $-14 + (-12) + 4$

Solution:

-22

Exercise:

Problem: $-17+(-18)+6$

Exercise:

Problem: $135+(-110)+83$

Solution:

108

Exercise:

Problem: $6 -38+27+(-8)+126$

Exercise:

Problem: $19+2\left(-3+8\right)$

Solution:

29

Exercise:

Problem: $24+3\left(-5+9\right)$

Subtract Integers

In the following exercises, simplify.

Exercise:

Problem: $8-2$

Solution:

6

Exercise:

Problem: $-6-(-4)$

Exercise:

Problem: $-5-4$

Solution:

-9

Exercise:

Problem: $-7-2$

Exercise:

Problem: $8-(-4)$

Solution:

12

Exercise:

Problem: $7-(-3)$

Exercise:

Problem: ⓐ $44-28$ ⓑ $44+(-28)$

Solution:

ⓐ 16 ⓑ 16

Exercise:

Problem: ⓐ $35-16$ ⓑ $35+(-16)$

Exercise:

Problem: ⓐ $27-(-18)$ ⓑ $27+18$

Solution:

ⓐ 45 ⓑ 45

Exercise:

Problem: ⓐ $46-(-37)$ ⓑ $46+37$

In the following exercises, simplify each expression.

Exercise:

Problem: $15 - (-12)$

Solution:

27

Exercise:

Problem: $14 - (-11)$

Exercise:

Problem: $48 - 87$

Solution:

-39

Exercise:

Problem: $45 - 69$

Exercise:

Problem: $-17 - 42$

Solution:

-59

Exercise:

Problem: $-19 - 46$

Exercise:

Problem: $-103 - (-52)$

Solution:

-51

Exercise:

Problem: $-105 - (-68)$

Exercise:

Problem: $-45 - (54)$

Solution:

9

Exercise:

Problem: $-58-(-67)$

Exercise:

Problem: $8-3-7$

Solution:

-2

Exercise:

Problem: $9-6-5$

Exercise:

Problem: $-5-4+7$

Solution:

-2

Exercise:

Problem: $-3-8+4$

Exercise:

Problem: $-14-(-27)+9$

Solution:

22

Exercise:

Problem: $64+(-17)-9$

Exercise:

Problem: $(2-7)-(3-8)(2)$

Solution:

0

Exercise:

Problem: $(1-8)-(2-9)$

Exercise:

Problem: $-(6-8)-(2-4)$

Solution:

4

Exercise:

Problem: $-(4-5)-(7-8)$

Exercise:

Problem: $25-[10-(3-12)]$

Solution:

6

Exercise:

Problem: $32-[5-(15-20)]$

Exercise:

Problem: $6.3-4.3-7.2$

Solution:

-8

Exercise:

Problem: $5.7-8.2-4.9$

Exercise:

Problem: 5^2-6^2

Solution:

-11

Exercise:

Problem: 6^2-7^2

Everyday Math

Exercise:

Problem:

Elevation The highest elevation in the United States is Mount McKinley, Alaska, at 20,320 feet above sea level. The lowest elevation is Death Valley, California, at 282 feet below sea level.

Use integers to write the elevation of:

ⓐ Mount McKinley.
ⓑ Death Valley.

Solution:

ⓐ 20,329 ⓑ -282

Exercise:

Problem:

Extreme temperatures The highest recorded temperature on Earth was 58° Celsius, recorded in the Sahara Desert in 1922. The lowest recorded temperature was 90° below 0° Celsius, recorded in Antarctica in 1983.

Use integers to write the:

ⓐ highest recorded temperature.
ⓑ lowest recorded temperature.

Exercise:

Problem:

State budgets In June, 2011, the state of Pennsylvania estimated it would have a budget surplus of $540 million. That same month, Texas estimated it would have a budget deficit of $27 billion.

Use integers to write the budget of:

ⓐ Pennsylvania.
ⓑ Texas.

Solution:

ⓐ $540 million ⓑ $-\$27$ billion

Exercise:

Problem:

College enrollments Across the United States, community college enrollment grew by 1,400,000 students from Fall 2007 to Fall 2010. In California, community college enrollment declined by 110,171 students from Fall 2009 to Fall 2010.

Use integers to write the change in enrollment:

ⓐ in the U.S. from Fall 2007 to Fall 2010.
ⓑ in California from Fall 2009 to Fall 2010.

Exercise:

Problem:

Stock Market The week of September 15, 2008 was one of the most volatile weeks ever for the US stock market. The closing numbers of the Dow Jones Industrial Average each day were:

Monday	-504
Tuesday	$+142$
Wednesday	-449
Thursday	$+410$
Friday	$+369$

What was the overall change for the week? Was it positive or negative?

Solution:

-32

Exercise:

Problem:

Stock Market During the week of June 22, 2009, the closing numbers of the Dow Jones Industrial Average each day were:

Monday	-201
Tuesday	-16
Wednesday	-23
Thursday	$+172$
Friday	-34

What was the overall change for the week? Was it positive or negative?

Writing Exercises

Exercise:

Problem: Give an example of a negative number from your life experience.

Solution:

Answers may vary

Exercise:

Problem: What are the three uses of the “ $-$ ” sign in algebra? Explain how they differ.

Exercise:

Problem: Explain why the sum of -8 and 2 is negative, but the sum of 8 and -2 is positive.

Solution:

Answers may vary

Exercise:

Problem: Give an example from your life experience of adding two negative numbers.

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
use negatives and opposites of integers.			
simplify expressions with absolute value.			
add integers.			
subtract integers.			

ⓑ What does this checklist tell you about your mastery of this section? What steps will you take to improve?

Glossary

absolute value
: The absolute value of a number is its distance from 0 on the number line. The absolute value of a number n is written as $|n|$.

integers
: The whole numbers and their opposites are called the integers: ...−3, −2, −1, 0, 1, 2, 3...

opposite

The opposite of a number is the number that is the same distance from zero on the number line but on the opposite side of zero: $-a$ means the opposite of the number. The notation $-a$ is read “the opposite of a.”

Multiply and Divide Integers
By the end of this section, you will be able to:

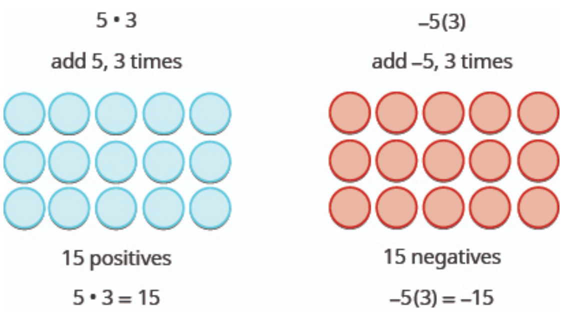
- Multiply integers
- Divide integers
- Simplify expressions with integers
- Evaluate variable expressions with integers
- Translate English phrases to algebraic expressions
- Use integers in applications

Note:
A more thorough introduction to the topics covered in this section can be found in the *Prealgebra* chapter, **Integers**.

Multiply Integers

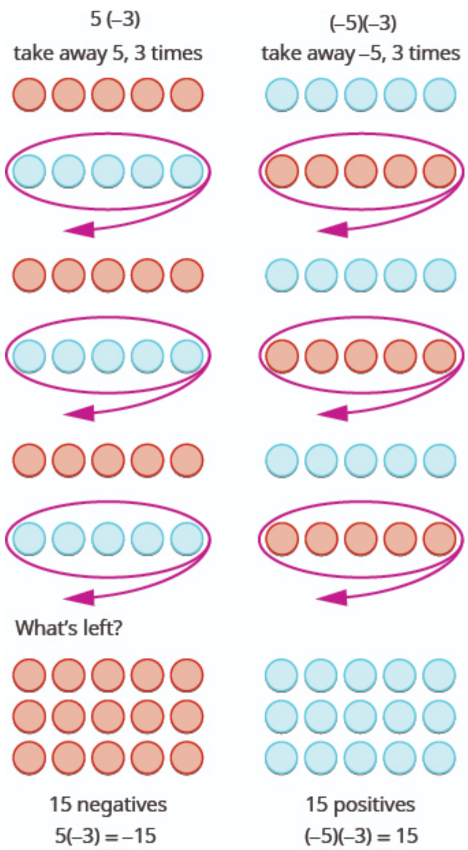
Since multiplication is mathematical shorthand for repeated addition, our model can easily be applied to show multiplication of integers. Let’s look at this concrete model to see what patterns we notice. We will use the same examples that we used for addition and subtraction. Here, we will use the model just to help us discover the pattern.

We remember that $a \cdot b$ means add a, b times. Here, we are using the model just to help us discover the pattern.



The next two examples are more interesting.

What does it mean to multiply 5 by -3? It means subtract 5, 3 times. Looking at subtraction as “taking away,” it means to take away 5, 3 times. But there is nothing to take away, so we start by adding neutral pairs on the workspace. Then we take away 5 three times.



In summary:
Equation:

$$\begin{array}{rclcrcl} 5 \cdot 3 & = & 15 & \quad & -5\,(3) & = & -15 \\ 5\,(-3) & = & -15 & \quad & (-5)\,(-3) & = & 15 \end{array}$$

Notice that for multiplication of two signed numbers, when the:

- signs are the *same*, the product is *positive*.
- signs are *different*, the product is *negative*.

We'll put this all together in the chart below.

Note:
Multiplication of Signed Numbers
For multiplication of two signed numbers:

Same signs	Product	Example

Same signs	Product	Example
Two positives Two negatives	Positive Positive	$7 \cdot 4 = 28$ $-8(-6) = 48$

Different signs	Product	Example
Positive · negative Negative · positive	Negative Negative	$7(-9) = -63$ $-5 \cdot 10 = -50$

Example:
Exercise:

Problem: Multiply: ⓐ $-9 \cdot 3$ ⓑ $-2(-5)$ ⓒ $4(-8)$ ⓓ $7 \cdot 6$.

Solution:
Solution

ⓐ

$$-9 \cdot 3$$

Multiply, noting that the signs are different so the product is negative.

$$-27$$

ⓑ

$$-2(-5)$$

Multiply, noting that the signs are the same so the product is positive.

$$10$$

ⓒ

$$4(-8)$$

Multiply, with different signs. -32

ⓓ

$$7 \cdot 6$$

Multiply, with same signs. 42

Note:
Exercise:

Problem: Multiply: ⓐ $-6 \cdot 8$ ⓑ $-4(-7)$ ⓒ $9(-7)$ ⓓ $5 \cdot 12$.

Solution:

ⓐ -48 ⓑ 28 ⓒ -63 ⓓ 60

Note:
Exercise:

Problem: Multiply: ⓐ $-8 \cdot 7$ ⓑ $-6\left(-9\right)$ ⓒ $7\left(-4\right)$ ⓓ $3 \cdot 13$.

Solution:

ⓐ -56 ⓑ 54 ⓒ -28 ⓓ 39

When we multiply a number by 1, the result is the same number. What happens when we multiply a number by -1? Let's multiply a positive number and then a negative number by -1 to see what we get.
Equation:

$$\begin{array}{lcc} & -1 \cdot 4 & -1(-3) \\ \text{Multiply.} & -4 & 3 \\ & -4 \text{ is the opposite of } 4. & 3 \text{ is the opposite of } -3. \end{array}$$

Each time we multiply a number by -1, we get its opposite!

Note:
Multiplication by -1
Equation:

$$-1a = -a$$

Multiplying a number by -1 gives its opposite.

Example:
Exercise:

Problem: Multiply: ⓐ $-1 \cdot 7$ ⓑ $-1\left(-11\right)$.

Solution:
Solution

ⓐ

$$\begin{array}{lc} & -1 \cdot 7 \\ \text{Multiply, noting that the signs are different} & -7 \\ \text{so the product is negative.} & -7 \text{ is the opposite of } 7. \end{array}$$

ⓑ

$$-1\,(-11)$$

Multiply, noting that the signs are the same so the product is positive. 11

11 is the opposite of -11.

Note:
Exercise:

Problem: Multiply: ⓐ $-1 \cdot 9$ ⓑ $-1 \cdot (-17)$.

Solution:

ⓐ -9 ⓑ 17

Note:
Exercise:

Problem: Multiply: ⓐ $-1 \cdot 8$ ⓑ $-1 \cdot (-16)$.

Solution:

ⓐ -8 ⓑ 16

Divide Integers

What about division? Division is the inverse operation of multiplication. So, $15 \div 3 = 5$ because $15 \cdot 3 = 5$. In words, this expression says that 15 can be divided into three groups of five each because adding five three times gives 15. Look at some examples of multiplying integers, to figure out the rules for dividing integers.
Equation:

$$\begin{array}{rclcrcl} 5 \cdot 3 & = & 15 \text{ so } 15 \div 3 & = & 5 & \qquad -5\,(3) & = & -15 \text{ so } -15 \div 3 & = & -5 \\ (-5)\,(-3) & = & 15 \text{ so } 15 \div (-3) & = & -5 & \qquad 5\,(-3) & = & -15 \text{ so } -15 \div (-3) & = & 5 \end{array}$$

Division follows the same rules as multiplication!

For division of two signed numbers, when the:

- signs are the *same*, the quotient is *positive*.
- signs are *different*, the quotient is *negative*.

And remember that we can always check the answer of a division problem by multiplying.

Note:
Multiplication and Division of Signed Numbers

For multiplication and division of two signed numbers:

- If the signs are the same, the result is positive.
- If the signs are different, the result is negative.

Same signs	**Result**
Two positives Two negatives	Positive Positive
If the signs are the same, the result is positive.	

Different signs	**Result**
Positive and negative Negative and positive	Negative Negative
If the signs are different, the result is negative.	

Example:
Exercise:

Problem: Divide: ⓐ $-27 \div 3$ ⓑ $-100 \div (-4)$.

Solution:
Solution

ⓐ

	$-27 \div 3$
Divide, with different signs, the quotient is negative.	-9

ⓑ

	$-100 \div (-4)$
Divide, with signs that are the same the quotient is positive.	25

Note:
Exercise:

Problem: Divide: ⓐ $-42 \div 6$ ⓑ $-117 \div (-3)$.

Solution:

ⓐ -7 ⓑ 39

Note:
Exercise:

Problem: Divide: ⓐ $-63 \div 7$ ⓑ $-115 \div (-5)$.

Solution:

ⓐ -9 ⓑ 23

Simplify Expressions with Integers

What happens when there are more than two numbers in an expression? The order of operations still applies when negatives are included. Remember My Dear Aunt Sally?

Let's try some examples. We'll simplify expressions that use all four operations with integers—addition, subtraction, multiplication, and division. Remember to follow the order of operations.

Example:
Exercise:

Problem: Simplify: $7(-2) + 4(-7) - 6$.

Solution:
Solution

	$7(-2) + 4(-7) - 6$
Multiply first.	$-14 + (-28) - 6$
Add.	$-42 - 6$
Subtract.	-48

Note:
Exercise:

Problem: Simplify: $8(-3) + 5(-7) - 4$.

Solution:

-63

Note:
Exercise:

Problem: Simplify: $9(-3)+7(-8)-1$.

Solution:

-84

Example:
Exercise:

Problem: Simplify: ⓐ $(-2)^4$ ⓑ -2^4.

Solution:
Solution

ⓐ

	$(-2)^4$
Write in expanded form.	$(-2)(-2)(-2)(-2)$
Multiply.	$4(-2)(-2)$
Multiply.	$-8(-2)$
Multiply.	16

ⓑ

	-2^4
Write in expanded form. We are asked to find the opposite of 2^4.	$-(2\cdot2\cdot2\cdot2)$
Multiply.	$-(4\cdot2\cdot2)$
Multiply.	$-(8\cdot2)$
Multiply.	-16

Notice the difference in parts ⓐ and ⓑ. In part ⓐ, the exponent means to raise what is in the parentheses, the (-2) to the 4^{th} power. In part ⓑ, the exponent means to raise just the 2 to the 4^{th} power and then take the opposite.

Note:
Exercise:

Problem: Simplify: ⓐ $(-3)^4$ ⓑ -3^4.

Solution:

ⓐ 81 ⓑ -81

Note:
Exercise:

Problem: Simplify: ⓐ $(-7)^2$ ⓑ -7^2.

Solution:

ⓐ 49 ⓑ -49

The next example reminds us to simplify inside parentheses first.

Example:
Exercise:

Problem: Simplify: $12 - 3(9 - 12)$.

Solution:
Solution

	$12 - 3(9 - 12)$
Subtract in parentheses first.	$12 - 3(-3)$
Multiply.	$12 - (-9)$
Subtract.	21

Note:
Exercise:

Problem: Simplify: $17 - 4(8 - 11)$.

Solution:

29

Note:
Exercise:

Problem: Simplify: $16 - 6\left(7 - 13\right)$.

Solution:

52

Example:
Exercise:

Problem: Simplify: $8\left(-9\right) \div \left(-2\right)^3$.

Solution:
Solution

	$8\left(-9\right) \div \left(-2\right)^3$
Exponents first.	$8\left(-9\right) \div \left(-8\right)$
Multiply.	$-72 \div \left(-8\right)$
Divide.	9

Note:
Exercise:

Problem: Simplify: $12\left(-9\right) \div \left(-3\right)^3$.

Solution:

4

Note:
Exercise:

Problem: Simplify: $18\left(-4\right) \div \left(-2\right)^3$.

Solution:

9

Example:
Exercise:

Problem: Simplify: $-30 \div 2 + \left(-3\right)\left(-7\right)$.

Solution:
Solution

	$-30 \div 2 + (-3)(-7)$
Multiply and divide left to right, so divide first.	$-15 + (-3)(-7)$
Multiply.	$-15 + 21$
Add.	6

Note:
Exercise:

Problem: Simplify: $-27 \div 3 + (-5)(-6)$.

Solution:

21

Note:
Exercise:

Problem: Simplify: $-32 \div 4 + (-2)(-7)$.

Solution:

6

Evaluate Variable Expressions with Integers

Remember that to evaluate an expression means to substitute a number for the variable in the expression. Now we can use negative numbers as well as positive numbers.

Example:
Exercise:

Problem: When $n = -5$, evaluate: ⓐ $n + 1$ ⓑ $-n + 1$.

Solution:
Solution

ⓐ

	$n + 1$
Substitute -5 for n.	$-5 + 1$
Simplify.	-4

ⓑ

	$-n + 1$
Substitute -5 for n.	$-(-5) + 1$
Simplify.	$5 + 1$
Add.	6

Note:
Exercise:

Problem: When $n = -8$, evaluate ⓐ $n + 2$ ⓑ $-n + 2$.

Solution:

ⓐ -6 ⓑ 10

Note:
Exercise:

Problem: When $y = -9$, evaluate ⓐ $y + 8$ ⓑ $-y + 8$.

Solution:

ⓐ -1 ⓑ 17

Example:
Exercise:

Problem: Evaluate $(x+y)^2$ when $x=-18$ and $y=24$.

Solution:
Solution

	$(x+y)^2$
Substitute −18 for x and 24 for y.	$(-18+24)^2$
Add inside parenthesis.	$(6)^2$
Simplify.	36

Note:
Exercise:

Problem: Evaluate $(x+y)^2$ when $x=-15$ and $y=29$.

Solution:

196

Note:
Exercise:

Problem: Evaluate $(x+y)^3$ when $x=-8$ and $y=10$.

Solution:

8

Example:
Exercise:

Problem: Evaluate $20 - z$ when ⓐ $z = 12$ and ⓑ $z = -12$.

Solution:
Solution

ⓐ

	$20 - z$
Substitute 12 for z.	$20 - 12$
Subtract.	8

ⓑ

	$20 - z$
Substitute -12 for z.	$20 - (-12)$
Subtract.	32

Note:
Exercise:

Problem: Evaluate: $17 - k$ when ⓐ $k = 19$ and ⓑ $k = -19$.

Solution:

ⓐ -2 ⓑ 36

Note:
Exercise:

Problem: Evaluate: $-5 - b$ when ⓐ $b = 14$ and ⓑ $b = -14$.

Solution:

ⓐ -19 ⓑ 9

Example:
Exercise:

Problem: Evaluate: $2x^2 + 3x + 8$ when $x = 4$.

Solution:
Solution

Substitute 4 for x. Use parentheses to show multiplication.

	$2x^2 + 3x + 8$
Substitute.	$2(4)^2 + 3(4) + 8$
Evaluate exponents.	$2(16) + 3(4) + 8$
Multiply.	$32 + 12 + 8$
Add.	52

Note:
Exercise:

Problem: Evaluate: $3x^2 - 2x + 6$ when $x = -3$.

Solution:

39

Note:
Exercise:

Problem: Evaluate: $4x^2 - x - 5$ when $x = -2$.

Solution:

13

Translate Phrases to Expressions with Integers

Our earlier work translating English to algebra also applies to phrases that include both positive and negative numbers.

Example:
Exercise:

Problem: Translate and simplify: the sum of 8 and -12, increased by 3.

Solution:
Solution

	$\text{the } \textbf{sum} \text{ of } 8 \text{ and } -12, \text{ increased by } 3$
Translate.	$[8 + (-12)] + 3$
Simplify. Be careful not to confuse the brackets with an absolute value sign.	$(-4) + 3$
Add.	-1

Note:
Exercise:

Problem: Translate and simplify the sum of 9 and -16, increased by 4.

Solution:

$(9 + (-16)) + 4 - 3$

Note:
Exercise:

Problem: Translate and simplify the sum of -8 and -12, increased by 7.

Solution:

$(-8+(-12))+7-13$

When we first introduced the operation symbols, we saw that the expression may be read in several ways. They are listed in the chart below.

$a-b$
a minus b the difference of a and b b subtracted from a b less than a

Be careful to get *a* and *b* in the right order!

Example:
Exercise:

Problem: Translate and then simplify ⓐ the difference of 13 and -21 ⓑ subtract 24 from -19.

Solution:
Solution

ⓐ

	the **difference** *of* 13 *and* -21
Translate.	$13-(-21)$
Simplify.	34

ⓑ

	subtract 24 **from** -19
Translate. Remember, "subtract b from a means $a-b$.	$-19-24$
Simplify.	-43

Note:
Exercise:

Problem: Translate and simplify ⓐ the difference of 14 and -23 ⓑ subtract 21 from -17.

Solution:

ⓐ $14-(-23); 37$ ⓑ $-17-21; -38$

Note:
Exercise:

Problem: Translate and simplify ⓐ the difference of 11 and -19 ⓑ subtract 18 from -11.

Solution:

ⓐ $11-(-19); 30$ ⓑ $-11-18; -29$

Once again, our prior work translating English to algebra transfers to phrases that include both multiplying and dividing integers. Remember that the key word for multiplication is "product" and for division is "quotient."

Example:
Exercise:

Problem: Translate to an algebraic expression and simplify if possible: the product of -2 and 14.

Solution:
Solution

$$\begin{array}{ll} & \text{the product } of\ -2\ and\ 14 \\ \text{Translate.} & (-2)\,(14) \\ \text{Simplify.} & -28 \end{array}$$

Note:
Exercise:

Problem: Translate to an algebraic expression and simplify if possible: the product of -5 and 12.

Solution:

$-5\,(12); -60$

Note:
Exercise:

Problem: Translate to an algebraic expression and simplify if possible: the product of 8 and -13.

Solution:

$-8\,(13); -104$

Example:
Exercise:

Problem: Translate to an algebraic expression and simplify if possible: the quotient of -56 and -7.

Solution:
Solution

	the quotient of -56 and -7
Translate.	$-56 \div (-7)$
Simplify.	8

Note:
Exercise:

Problem: Translate to an algebraic expression and simplify if possible: the quotient of -63 and -9.

Solution:

$-63 \div (-9); 7$

Note:
Exercise:

Problem: Translate to an algebraic expression and simplify if possible: the quotient of -72 and -9.

Solution:

$-72 \div (-9); 8$

Use Integers in Applications

We'll outline a plan to solve applications. It's hard to find something if we don't know what we're looking for or what to call it! So when we solve an application, we first need to determine what the problem is asking us to find. Then we'll write a phrase that gives the information to find it. We'll translate the phrase into an expression and then simplify the expression to get the answer. Finally, we summarize the answer in a sentence to make sure it makes sense.

Example:
How to Apply a Strategy to Solve Applications with Integers

Exercise:

Problem:

The temperature in Urbana, Illinois one morning was 11 degrees. By mid-afternoon, the temperature had dropped to -9 degrees. What was the difference of the morning and afternoon temperatures?

Solution:
Solution

Step 1. Read the problem. Make sure all the words and ideas are understood.	
Step 2. Identify what we are asked to find.	the difference of the morning and afternoon temperatures
Step 3. Write a phrase the gives the information to find it.	the *difference of* 11 *and* –9
Step 4. Translate the phrase to an expression.	$11-(-9)$
Step 5. Simplify the expression.	20
Step 6. Write a complete sentence that answers the question.	The difference in temperatures was 20 degrees.

Note:
Exercise:

Problem:

The temperature in Anchorage, Alaska one morning was 15 degrees. By mid-afternoon the temperature had dropped to 30 degrees below zero. What was the difference in the morning and afternoon temperatures?

Solution:

The difference in temperatures was 45 degrees.

Note:
Exercise:

Problem:

The temperature in Denver was -6 degrees at lunchtime. By sunset the temperature had dropped to -15 degrees. What was the difference in the lunchtime and sunset temperatures?

Solution:

The difference in temperatures was 9 degrees.

Note:
Apply a Strategy to Solve Applications with Integers.

Read the problem. Make sure all the words and ideas are understood
Identify what we are asked to find.
Write a phrase that gives the information to find it.
Translate the phrase to an expression.
Simplify the expression.
Answer the question with a complete sentence.

Example:
Exercise:

Problem:

The Mustangs football team received three penalties in the third quarter. Each penalty gave them a loss of fifteen yards. What is the number of yards lost?

Solution:
Solution

Step 1. Read the problem. Make sure all the words and ideas are understood.	
Step 2. Identify what we are asked to find.	the number of yards lost
Step 3. Write a phrase that gives the information to find it.	three times a 15-yard penalty
Step 4. Translate the phrase to an expression.	$3\left(-15\right)$
Step 5. Simplify the expression.	-45
Step 6. Answer the question with a complete sentence.	The team lost 45 yards.

Note:
Exercise:

Problem:

The Bears played poorly and had seven penalties in the game. Each penalty resulted in a loss of 15 yards. What is the number of yards lost due to penalties?

Solution:

The Bears lost 105 yards.

Note:
Exercise:

Problem:

Bill uses the ATM on campus because it is convenient. However, each time he uses it he is charged a $2 fee. Last month he used the ATM eight times. How much was his total fee for using the ATM?

Solution:

A $16 fee was deducted from his checking account.

Key Concepts

- **Multiplication and Division of Two Signed Numbers**
  - Same signs—Product is positive
  - Different signs—Product is negative
- **Strategy for Applications**

  Identify what you are asked to find.
  Write a phrase that gives the information to find it.
  Translate the phrase to an expression.
  Simplify the expression.
  Answer the question with a complete sentence.

Practice Makes Perfect

Multiply Integers

In the following exercises, multiply.
Exercise:

Problem: $-4 \cdot 8$

Solution:

-32

Exercise:

Problem: $-3 \cdot 9$

Exercise:

Problem: $9\left(-7\right)$

Solution:

-63

Exercise:

Problem: $13\left(-5\right)$

Exercise:

Problem: -1.6

Solution:

-6

Exercise:

Problem: -1.3

Exercise:

Problem: $-1\left(-14\right)$

Solution:

14

Exercise:

Problem: $-1\left(-19\right)$

Divide Integers

In the following exercises, divide.

Exercise:

Problem: $-24 \div 6$

Solution:

-4

Exercise:

Problem: $35 \div \left(-7\right)$

Exercise:

Problem: $-52 \div \left(-4\right)$

Solution:

13

Exercise:

Problem: $-84 \div (-6)$

Exercise:

Problem: $-180 \div 15$

Solution:

-12

Exercise:

Problem: $-192 \div 12$

Simplify Expressions with Integers

In the following exercises, simplify each expression.

Exercise:

Problem: $5\left(-6\right)+7\left(-2\right)-3$

Solution:

-47

Exercise:

Problem: $8\left(-4\right)+5\left(-4\right)-6$

Exercise:

Problem: $\left(-2\right)^{6}$

Solution:

64

Exercise:

Problem: $\left(-3\right)^{5}$

Exercise:

Problem: -4^{2}

Solution:

-16

Exercise:

Problem: -6^{2}

Exercise:

Problem: $-3\,(-5)\,(6)$

Solution:

90

Exercise:

Problem: $-4\,(-6)\,(3)$

Exercise:

Problem: $(8-11)\,(9-12)$

Solution:

9

Exercise:

Problem: $(6-11)\,(8-13)$

Exercise:

Problem: $26-3\,(2-7)$

Solution:

41

Exercise:

Problem: $23-2\,(4-6)$

Exercise:

Problem: $65\div(-5)+(-28)\div(-7)$

Solution:

-9

Exercise:

Problem: $52\div(-4)+(-32)\div(-8)$

Exercise:

Problem: $9-2\,[3-8\,(-2)]$

Solution:

-29

Exercise:

Problem: $11 - 3\left[7 - 4\left(-2\right)\right]$

Exercise:

Problem: $\left(-3\right)^2 - 24 \div \left(8 - 2\right)$

Solution:

5

Exercise:

Problem: $\left(-4\right)^2 - 32 \div \left(12 - 4\right)$

Evaluate Variable Expressions with Integers

In the following exercises, evaluate each expression.

Exercise:

Problem: $y + \left(-14\right)$ when
ⓐ $y = -33$
ⓑ $y = 30$

Solution:

ⓐ -47 ⓑ 16

Exercise:

Problem: $x + \left(-21\right)$ when
ⓐ $x = -27$
ⓑ $x = 44$

Exercise:

Problem: ⓐ $a + 3$ when $a = -7$
ⓑ $-a + 3$ when $a = -7$

Solution:

ⓐ -4 ⓑ 10

Exercise:

Problem: ⓐ $d + \left(-9\right)$ when $d = -8$
ⓑ $-d + \left(-9\right)$ when $d = -8$

Exercise:

Problem: $m + n$ when
$m = -15, n = 7$

Solution:

-8

Exercise:

Problem: $p + q$ when $p = -9, q = 17$

Exercise:

Problem: $r + s$ when $r = -9, s = -7$

Solution:

-16

Exercise:

Problem: $t + u$ when $t = -6, u = -5$

Exercise:

Problem: $(x + y)^2$ when $x = -3, y = 14$

Solution:

121

Exercise:

Problem: $(y + z)^2$ when $y = -3, z = 15$

Exercise:

Problem: $-2x + 17$ when
ⓐ $x = 8$
ⓑ $x = -8$

Solution:

ⓐ 1 ⓑ 33

Exercise:

Problem: $-5y + 14$ when
ⓐ $y = 9$
ⓑ $y = -9$

Exercise:

Problem: $10 - 3m$ when
ⓐ $m = 5$
ⓑ $m = -5$

Solution:

ⓐ -5 ⓑ 25

Exercise:

Problem: $18 - 4n$ when
ⓐ $n = 3$
ⓑ $n = -3$

Exercise:

Problem: $2w^2 - 3w + 7$ when
$w = -2$

Solution:

21

Exercise:

Problem: $3u^2 - 4u + 5$ when $u = -3$

Exercise:

Problem: $9a - 2b - 8$ when
$a = -6$ and $b = -3$

Solution:

-56

Exercise:

Problem: $7m - 4n - 2$ when
$m = -4$ and $n = -9$

Translate English Phrases to Algebraic Expressions

In the following exercises, translate to an algebraic expression and simplify if possible.

Exercise:

Problem: the sum of 3 and -15, increased by 7

Solution:

$(3 + (-15)) + 7; -5$

Exercise:

Problem: the sum of -8 and -9, increased by 23

Exercise:

Problem: the difference of 10 and -18

Solution:

$10-(-18); 28$

Exercise:

Problem: subtract 11 from -25

Exercise:

Problem: the difference of -5 and -30

Solution:

$-5-(-30); 25$

Exercise:

Problem: subtract -6 from -13

Exercise:

Problem: the product of -3 and 15

Solution:

$-3 \cdot 15; -45$

Exercise:

Problem: the product of -4 and 16

Exercise:

Problem: the quotient of -60 and -20

Solution:

$-60 \div (-20); 3$

Exercise:

Problem: the quotient of -40 and -20

Exercise:

Problem: the quotient of -6 and the sum of *a* and *b*

Solution:

$\frac{-6}{a+b}$

Exercise:

Problem: the quotient of -7 and the sum of *m* and *n*

Exercise:

Problem: the product of -10 and the difference of p and q

Solution:

$-10\left(p-q\right)$

Exercise:

Problem: the product of -13 and the difference of c and d

Use Integers in Applications

In the following exercises, solve.

Exercise:

Problem:

Temperature On January 15, the high temperature in Anaheim, California, was 84°. That same day, the high temperature in Embarrass, Minnesota was -12°. What was the difference between the temperature in Anaheim and the temperature in Embarrass?

Solution:

96°

Exercise:

Problem:

Temperature On January 21, the high temperature in Palm Springs, California, was 89°, and the high temperature in Whitefield, New Hampshire was -31°. What was the difference between the temperature in Palm Springs and the temperature in Whitefield?

Exercise:

Problem:

Football At the first down, the Chargers had the ball on their 25 yard line. On the next three downs, they lost 6 yards, gained 10 yards, and lost 8 yards. What was the yard line at the end of the fourth down?

Solution:

21

Exercise:

Problem:

Football At the first down, the Steelers had the ball on their 30 yard line. On the next three downs, they gained 9 yards, lost 14 yards, and lost 2 yards. What was the yard line at the end of the fourth down?

Exercise:

Problem:

Checking Account Mayra has $124 in her checking account. She writes a check for $152. What is the new balance in her checking account?

Solution:

$-\$28$

Exercise:

Problem:

Checking Account Selina has \$165 in her checking account. She writes a check for \$207. What is the new balance in her checking account?

Exercise:

Problem:

Checking Account Diontre has a balance of $-\$38$ in his checking account. He deposits \$225 to the account. What is the new balance?

Solution:

\$187

Exercise:

Problem:

Checking Account Reymonte has a balance of $-\$49$ in his checking account. He deposits \$281 to the account. What is the new balance?

Everyday Math

Exercise:

Problem:

Stock market Javier owns 300 shares of stock in one company. On Tuesday, the stock price dropped \$12 per share. What was the total effect on Javier's portfolio?

Solution:

$-\$3600$

Exercise:

Problem:

Weight loss In the first week of a diet program, eight women lost an average of 3 pounds each. What was the total weight change for the eight women?

Writing Exercises

Exercise:

Problem: In your own words, state the rules for multiplying integers.

Solution:

Answers may vary

Exercise:

Problem: In your own words, state the rules for dividing integers.

Exercise:

Problem: Why is $-2^4 \neq (-2)^4$?

Solution:

Answers may vary

Exercise:

Problem: Why is $-4^3 = (-4)^3$?

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
multiply integers.			
divide integers.			
simplify expressions with integers.			
evaluate variable expressions with integers.			
translate English phrases to algebraic expressions.			
use integers in applications.			

ⓑ On a scale of 1–10, how would you rate your mastery of this section in light of your responses on the checklist? How can you improve this?

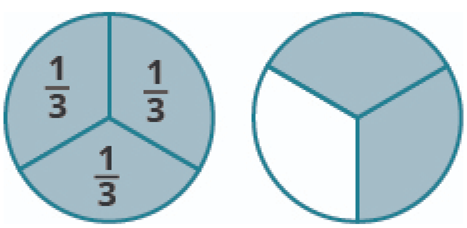
Visualize Fractions
By the end of this section, you will be able to:

- Find equivalent fractions
- Simplify fractions
- Multiply fractions
- Divide fractions
- Simplify expressions written with a fraction bar
- Translate phrases to expressions with fractions

Note:
A more thorough introduction to the topics covered in this section can be found in the *Prealgebra* chapter, **Fractions**.

Find Equivalent Fractions

Fractions are a way to represent parts of a whole. The fraction $\frac{1}{3}$ means that one whole has been divided into 3 equal parts and each part is one of the three equal parts. See [link]. The fraction $\frac{2}{3}$ represents two of three equal parts. In the fraction $\frac{2}{3}$, the 2 is called the **numerator** and the 3 is called the **denominator**.



The circle on the left has been divided into 3 equal parts. Each part is $\frac{1}{3}$ of the 3 equal

parts. In the circle on the right, $\frac{2}{3}$ of the circle is shaded (2 of the 3 equal parts).

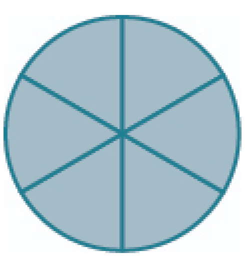
Note: Doing the Manipulative Mathematics activity “Model Fractions” will help you develop a better understanding of fractions, their numerators and denominators.

Note:
Fraction
A **fraction** is written $\frac{a}{b}$, where $b \neq 0$ and

- a is the **numerator** and b is the **denominator**.

A fraction represents parts of a whole. The denominator b is the number of equal parts the whole has been divided into, and the numerator a indicates how many parts are included.

If a whole pie has been cut into 6 pieces and we eat all 6 pieces, we ate $\frac{6}{6}$ pieces, or, in other words, one whole pie.



So $\frac{6}{6} = 1$. This leads us to the property of one that tells us that any number, except zero, divided by itself is 1.

Note:
Property of One
Equation:

$$\frac{a}{a} = 1 \qquad (a \neq 0)$$

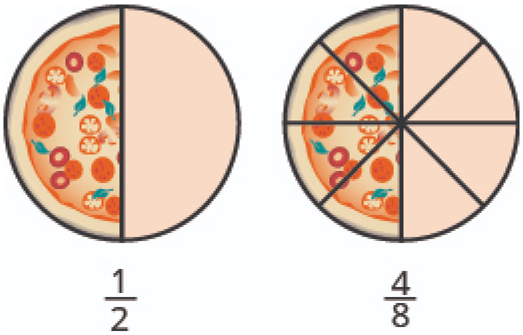
Any number, except zero, divided by itself is one.

Note: Doing the Manipulative Mathematics activity “Fractions Equivalent to One” will help you develop a better understanding of fractions that are equivalent to one.

If a pie was cut in 6 pieces and we ate all 6, we ate $\frac{6}{6}$ pieces, or, in other words, one whole pie. If the pie was cut into 8 pieces and we ate all 8, we ate $\frac{8}{8}$ pieces, or one whole pie. We ate the same amount—one whole pie.

The fractions $\frac{6}{6}$ and $\frac{8}{8}$ have the same value, 1, and so they are called equivalent fractions. **Equivalent fractions** are fractions that have the same value.

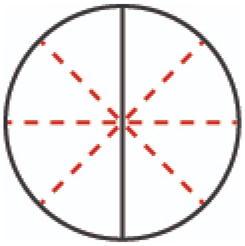
Let’s think of pizzas this time. [link] shows two images: a single pizza on the left, cut into two equal pieces, and a second pizza of the same size, cut into eight pieces on the right. This is a way to show that $\frac{1}{2}$ is equivalent to $\frac{4}{8}$. In other words, they are equivalent fractions.



Since the same amount is of each pizza is shaded, we see that $\frac{1}{2}$ is equivalent to $\frac{4}{8}$. They are equivalent fractions.

Note:
Equivalent Fractions
Equivalent fractions are fractions that have the same value.

How can we use mathematics to change $\frac{1}{2}$ into $\frac{4}{8}$? How could we take a pizza that is cut into 2 pieces and cut it into 8 pieces? We could cut each of the 2 larger pieces into 4 smaller pieces! The whole pizza would then be cut into 8 pieces instead of just 2. Mathematically, what we've described could be written like this as $\frac{1 \cdot 4}{2 \cdot 4} = \frac{4}{8}$. See [link].



Cutting each half of the pizza into 4 pieces, gives us pizza cut into 8 pieces: $\frac{1 \cdot 4}{2 \cdot 4} = \frac{4}{8}$.

This model leads to the following property:

Note:
Equivalent Fractions Property
If a, b, c are numbers where $b \neq 0, c \neq 0$, then
Equation:

$$\frac{a}{b} = \frac{a \cdot c}{b \cdot c}$$

If we had cut the pizza differently, we could get

$$\frac{1 \cdot 2}{2 \cdot 2} = \frac{2}{4} \quad \text{so} \quad \frac{1}{2} = \frac{2}{4}$$

$$\frac{1 \cdot 3}{2 \cdot 3} = \frac{3}{6} \quad \text{so} \quad \frac{1}{2} = \frac{3}{6}$$

$$\frac{1 \cdot 10}{2 \cdot 10} = \frac{10}{20} \quad \text{so} \quad \frac{1}{2} = \frac{10}{20}$$

So, we say $\frac{1}{2}$, $\frac{2}{4}$, $\frac{3}{6}$, and $\frac{10}{20}$ are equivalent fractions.

Note: Doing the Manipulative Mathematics activity "Equivalent Fractions" will help you develop a better understanding of what it means when two fractions are equivalent.

Example:
Exercise:

Problem: Find three fractions equivalent to $\frac{2}{5}$.

Solution:
Solution

To find a fraction equivalent to $\frac{2}{5}$, we multiply the numerator and denominator by the same number. We can choose any number, except for zero. Let's multiply them by 2, 3, and then 5.

$$\frac{2 \cdot 2}{5 \cdot 2} = \frac{4}{10} \qquad \frac{2 \cdot 3}{5 \cdot 3} = \frac{6}{15} \qquad \frac{2 \cdot 5}{5 \cdot 5} = \frac{10}{25}$$

So, $\frac{4}{10}$, $\frac{6}{15}$, and $\frac{10}{25}$ are equivalent to $\frac{2}{5}$.

Note:
Exercise:

Problem: Find three fractions equivalent to $\frac{3}{5}$.

Solution:

$\frac{6}{10}$, $\frac{9}{15}$, $\frac{12}{20}$; answers may vary

Note:
Exercise:

Problem: Find three fractions equivalent to $\frac{4}{5}$.

Solution:

$\frac{8}{10}$, $\frac{12}{15}$, $\frac{16}{20}$; answers may vary

Simplify Fractions

A fraction is considered **simplified** if there are no common factors, other than 1, in its numerator and denominator.

For example,

- $\frac{2}{3}$ is simplified because there are no common factors of 2 and 3.
- $\frac{10}{15}$ is not simplified because 5 is a common factor of 10 and 15.

Note:

Simplified Fraction

A fraction is considered **simplified** if there are no common factors in its numerator and denominator.

The phrase *reduce a fraction* means to simplify the fraction. We simplify, or reduce, a fraction by removing the common factors of the numerator and denominator. A fraction is not simplified until all common factors have been removed. If an expression has fractions, it is not completely simplified until the fractions are simplified.

In [link], we used the equivalent fractions property to find equivalent fractions. Now we'll use the equivalent fractions property in reverse to simplify fractions. We can rewrite the property to show both forms together.

Note:

Equivalent Fractions Property

If a, b, c are numbers where $b \neq 0, c \neq 0$,

Equation:

$$\text{then} \quad \frac{a}{b} = \frac{a \cdot c}{b \cdot c} \quad \text{and} \quad \frac{a \cdot c}{b \cdot c} = \frac{a}{b}$$

Example:

Exercise:

Problem: Simplify: $-\frac{32}{56}$.

Solution:

Solution

	$-\frac{32}{56}$
Rewrite the numerator and denominator showing the common factors.	$-\frac{4 \cdot 8}{7 \cdot 8}$
Simplify using the equivalent fractions property.	$-\frac{4}{7}$

Notice that the fraction $-\frac{4}{7}$ is simplified because there are no more common factors.

Note:
Exercise:

Problem: Simplify: $-\frac{42}{54}$.

Solution:

$-\frac{7}{9}$

Note:
Exercise:

Problem: Simplify: $-\frac{45}{81}$.

Solution:

$-\frac{5}{9}$

Sometimes it may not be easy to find common factors of the numerator and denominator. When this happens, a good idea is to factor the numerator and the denominator into prime numbers. Then divide out the common factors using the equivalent fractions property.

Example:
How to Simplify a Fraction
Exercise:

Problem: Simplify: $-\frac{210}{385}$.

Solution:
Solution

Step 1. Rewrite the numerator and denominator to show the common factors. If needed, factor the numerator and denominator into prime numbers first.	Rewrite 210 and 385 as the product of the primes.	$-\frac{210}{385}$ $-\frac{2\cdot3\cdot5\cdot7}{5\cdot7\cdot11}$
Step 2. Simplify using the equivalent fractions property by dividing out common factors.	Mark the common factors 5 and 7. Divide out the common factors.	$-\frac{2\cdot3\cdot\cancel{5}\cdot\cancel{7}}{\cancel{5}\cdot\cancel{7}\cdot11}$ $-\frac{2\cdot3}{11}$
Step 3. Multiply the remaining factors, if necessary.		$-\frac{6}{11}$

Note:
Exercise:

Problem: Simplify: $-\frac{69}{120}$.

Solution:

$-\frac{23}{40}$

Note:
Exercise:

Problem: Simplify: $-\frac{120}{192}$.

Solution:

$-\frac{5}{8}$

We now summarize the steps you should follow to simplify fractions.

Note:
Simplify a Fraction.

Rewrite the numerator and denominator to show the common factors.	If needed, factor the numerator and denominator into prime numbers first.

Simplify using the equivalent fractions property by dividing out common factors.
Multiply any remaining factors, if needed.

Example:
Exercise:

Problem: Simplify: $\frac{5x}{5y}$.

Solution:
Solution

	$\frac{5x}{5y}$
Rewrite showing the common factors, then divide out the common factors.	$\frac{\cancel{5} \cdot x}{\cancel{5} \cdot y}$
Simplify.	$\frac{x}{y}$

Note:
Exercise:

Problem: Simplify: $\frac{7x}{7y}$.

Solution:

$\frac{x}{y}$

Note:

Exercise:

Problem: Simplify: $\frac{3a}{3b}$.

Solution:

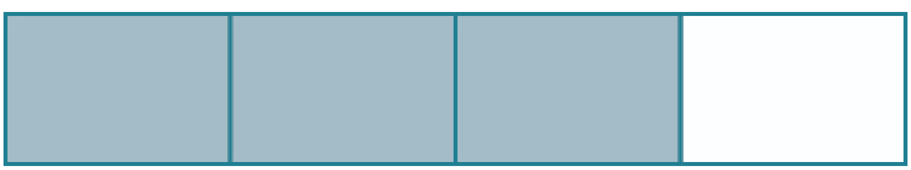
$\frac{a}{b}$

Multiply Fractions

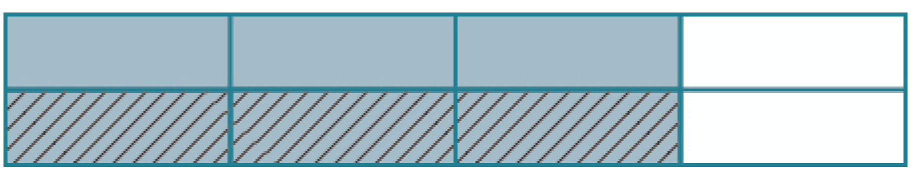
Many people find multiplying and dividing fractions easier than adding and subtracting fractions. So we will start with fraction multiplication.

Note: Doing the Manipulative Mathematics activity "Model Fraction Multiplication" will help you develop a better understanding of multiplying fractions.

We'll use a model to show you how to multiply two fractions and to help you remember the procedure. Let's start with $\frac{3}{4}$.



Now we'll take $\frac{1}{2}$ of $\frac{3}{4}$.



Notice that now, the whole is divided into 8 equal parts. So $\frac{1}{2} \cdot \frac{3}{4} = \frac{3}{8}$.

To multiply fractions, we multiply the numerators and multiply the denominators.

Note:
Fraction Multiplication
If a, b, c and d are numbers where $b \neq 0$ and $d \neq 0$, then
Equation:

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$$

To multiply fractions, multiply the numerators and multiply the denominators.

When multiplying fractions, the properties of positive and negative numbers still apply, of course. It is a good idea to determine the sign of the product as the first step. In [link], we will multiply negative and a positive, so the product will be negative.

Example:
Exercise:

Problem: Multiply: $-\frac{11}{12} \cdot \frac{5}{7}$.

Solution:
Solution

The first step is to find the sign of the product. Since the signs are the different, the product is negative.

	$-\frac{11}{12}\cdot\frac{5}{7}$
Determine the sign of the product; multiply.	$-\frac{11\cdot 5}{12\cdot 7}$
Are there any common factors in the numerator and the denominator? No	$-\frac{55}{84}$

Note:
Exercise:

Problem: Multiply: $-\frac{10}{28}\cdot\frac{8}{15}$.

Solution:

$-\frac{4}{21}$

Note:
Exercise:

Problem: Multiply: $-\frac{9}{20}\cdot\frac{5}{12}$.

Solution:

$-\frac{3}{16}$

When multiplying a fraction by an integer, it may be helpful to write the integer as a fraction. Any integer, a, can be written as $\frac{a}{1}$. So, for example, $3=\frac{3}{1}$.

Example:
Exercise:

Problem: Multiply: $-\frac{12}{5}(-20x)$.

Solution:
Solution

Determine the sign of the product. The signs are the same, so the product is positive.

	$-\frac{12}{5}(-20x)$
Write $20x$ as a fraction.	$\frac{12}{5}\left(\frac{20x}{1}\right)$
Multiply.	
Rewrite 20 to show the common factor 5 and divide it out.	$\frac{12\cdot 4\cdot \cancel{5}x}{\cancel{5}\cdot 1}$
Simplify.	$48x$

Note:
Exercise:

Problem: Multiply: $\frac{11}{3}(-9a)$.

Solution:

$-33a$

Note:
Exercise:

Problem: Multiply: $\frac{13}{7}(-14b)$.

Solution:

$-36b$

Divide Fractions

Now that we know how to multiply fractions, we are almost ready to divide. Before we can do that, that we need some vocabulary.

The **reciprocal** of a fraction is found by inverting the fraction, placing the numerator in the denominator and the denominator in the numerator. The reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$.

Notice that $\frac{2}{3} \cdot \frac{3}{2} = 1$. A number and its reciprocal multiply to 1.

To get a product of positive 1 when multiplying two numbers, the numbers must have the same sign. So reciprocals must have the same sign.

The reciprocal of $-\frac{10}{7}$ is $-\frac{7}{10}$, since $-\frac{10}{7}\left(-\frac{7}{10}\right) = 1$.

Note:

Reciprocal

The **reciprocal** of $\frac{a}{b}$ is $\frac{b}{a}$.

A number and its reciprocal multiply to one $\frac{a}{b} \cdot \frac{b}{a} = 1$.

Note: Doing the Manipulative Mathematics activity "Model Fraction Division" will help you develop a better understanding of dividing fractions.

To divide fractions, we multiply the first fraction by the reciprocal of the second.

Note:

Fraction Division

If a, b, c and d are numbers where $b \neq 0, c \neq 0$ and $d \neq 0$, then

Equation:

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$$

To divide fractions, we multiply the first fraction by the reciprocal of the second.

We need to say $b \neq 0, c \neq 0$ and $d \neq 0$ to be sure we don't divide by zero!

Example:

Exercise:

Problem: Divide: $-\frac{2}{3} \div \frac{n}{5}$.

Solution:
Solution

	$-\frac{2}{3} \div \frac{n}{5}$
To divide, multiply the first fraction by the reciprocal of the second.	$-\frac{2}{3} \cdot \frac{5}{n}$
Multiply.	$-\frac{10}{3n}$

Note:
Exercise:

Problem: Divide: $-\frac{3}{5} \div \frac{p}{7}$.

Solution:

$-\frac{21}{5p}$

Note:
Exercise:

Problem: Divide: $-\frac{5}{8} \div \frac{q}{3}$.

Solution:

$-\frac{15}{8q}$

Example:

Exercise:

Problem: Find the quotient: $-\frac{7}{8} \div \left(-\frac{14}{27}\right)$.

Solution:
Solution

	$-\frac{7}{18} \div \left(-\frac{14}{27}\right)$
To divide, multiply the first fraction by the reciprocal of the second.	$-\frac{7}{18} \cdot -\frac{27}{14}$
Determine the sign of the product, and then multiply..	$\frac{7 \cdot 27}{18 \cdot 14}$
Rewrite showing common factors.	$\frac{\cancel{7} \cdot \cancel{9} \cdot 3}{\cancel{9} \cdot 2 \cdot \cancel{7} \cdot 2}$
Remove common factors.	$\frac{3}{2 \cdot 2}$
Simplify.	$\frac{3}{4}$

Note:
Exercise:

Problem: Find the quotient: $-\frac{7}{27} \div \left(-\frac{35}{36}\right)$.

Solution:

$\frac{4}{15}$

Note:
Exercise:

Problem: Find the quotient: $-\frac{5}{14} \div \left(-\frac{15}{28}\right)$.

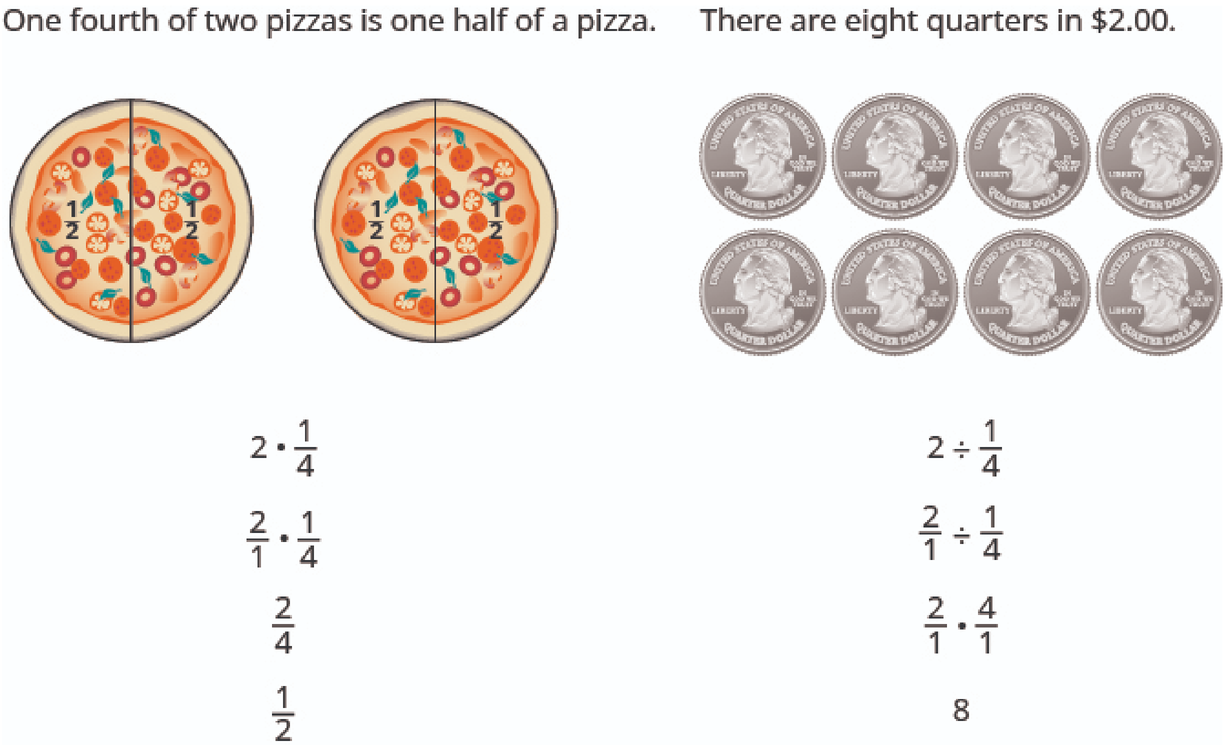
Solution:

$\frac{2}{3}$

There are several ways to remember which steps to take to multiply or divide fractions. One way is to repeat the call outs to yourself. If you do this each time you do an exercise, you will have the steps memorized.

- “To multiply fractions, multiply the numerators and multiply the denominators.”
- “To divide fractions, multiply the first fraction by the reciprocal of the second.”

Another way is to keep two examples in mind:



The numerators or denominators of some fractions contain fractions themselves. A fraction in which the numerator or the denominator is a fraction is called a **complex fraction**.

> **Note:**
> Complex Fraction
> A **complex fraction** is a fraction in which the numerator or the denominator contains a fraction.

Some examples of complex fractions are:

Equation:

$$\frac{\frac{6}{7}}{3} \quad \frac{\frac{3}{4}}{\frac{5}{8}} \quad \frac{\frac{x}{2}}{\frac{5}{6}}$$

To simplify a complex fraction, we remember that the fraction bar means division. For example, the complex fraction $\frac{\frac{3}{4}}{\frac{5}{8}}$ means $\frac{3}{4} \div \frac{5}{8}$.

Example:
Exercise:

Problem: Simplify: $\frac{\frac{3}{4}}{\frac{5}{8}}$.

Solution:
Solution

	$\frac{\frac{3}{4}}{\frac{5}{8}}$
Rewrite as division.	$\frac{3}{4} \div \frac{5}{8}$
Multiply the first fraction by the reciprocal of the second.	$\frac{3}{4} \cdot \frac{8}{5}$
Multiply.	$\frac{3 \cdot 8}{4 \cdot 5}$
Look for common factors.	$\frac{3 \cdot \cancel{4} \cdot 2}{\cancel{4} \cdot 5}$
Divide out common factors and simplify.	$\frac{6}{5}$

Note:
Exercise:

Problem: Simplify: $\frac{\frac{2}{3}}{\frac{5}{6}}$.

Solution:

$\frac{4}{5}$

Note:
Exercise:

Problem: Simplify: $\frac{\frac{3}{7}}{\frac{6}{11}}$.

Solution:

$\frac{11}{14}$

Example:
Exercise:

Problem: Simplify: $\frac{\frac{x}{2}}{\frac{xy}{6}}$.

Solution:
Solution

	$\frac{\frac{x}{2}}{\frac{xy}{6}}$
Rewrite as division.	$\frac{x}{2} \div \frac{xy}{6}$
Multiply the first fraction by the reciprocal of the second.	$\frac{x}{2} \cdot \frac{6}{xy}$
Multiply.	$\frac{x \cdot 6}{2 \cdot xy}$
Look for common factors.	$\frac{\cancel{x} \cdot 3 \cdot \cancel{2}}{\cancel{2} \cdot \cancel{x} \cdot y}$
Divide out common factors and simplify.	$\frac{3}{y}$

Note:
Exercise:

Problem: Simplify: $\frac{\frac{a}{8}}{\frac{ab}{6}}$.

Solution:

$\frac{3}{4b}$

Note:
Exercise:

Problem: Simplify: $\frac{\frac{p}{2}}{\frac{pq}{8}}$.

Solution:

$\frac{4}{2q}$

Simplify Expressions with a Fraction Bar

The line that separates the numerator from the denominator in a fraction is called a fraction bar. A fraction bar acts as grouping symbol. The order of operations then tells us to simplify the numerator and then the denominator. Then we divide.

To simplify the expression $\frac{5-3}{7+1}$, we first simplify the numerator and the denominator separately. Then we divide.

Equation:

$$\frac{5-3}{7+1}$$

Equation:

$$\frac{2}{8}$$

Equation:

$$\frac{1}{4}$$

Note:

Simplify an Expression with a Fraction Bar.

Simplify the expression in the numerator. Simplify the expression in the denominator.
Simplify the fraction.

Example:
Exercise:

Problem: Simplify: $\frac{4-2(3)}{2^2+2}$.

Solution:
Solution

	$\frac{4-2(3)}{2^2+2}$
Use the order of operations to simplify the numerator and the denominator.	$\frac{4-6}{4+2}$
Simplify the numerator and the denominator.	$\frac{-2}{6}$
Simplify. A negative divided by a positive is negative.	$-\frac{1}{3}$

Note:
Exercise:

Problem: Simplify: $\frac{6-3(5)}{3^2+3}$.

Solution:

$-\frac{3}{4}$

Note:
Exercise:

Problem: Simplify: $\frac{4-4(6)}{3^2+3}$.

Solution:

$-\frac{2}{3}$

Where does the negative sign go in a fraction? Usually the negative sign is in front of the fraction, but you will sometimes see a fraction with a negative numerator, or sometimes with a negative denominator. Remember that fractions represent division. When the numerator and denominator have different signs, the quotient is negative.

Equation:

$$\begin{array}{lll}\frac{-1}{3}=-\frac{1}{3} & \qquad & \frac{\text{negative}}{\text{positive}}=\text{negative} \\ \frac{1}{-3}=-\frac{1}{3} & \qquad & \frac{\text{positive}}{\text{negative}}=\text{negative}\end{array}$$

Note:
Placement of Negative Sign in a Fraction
For any positive numbers *a* and *b*,
Equation:

$$\frac{-a}{b}=\frac{a}{-b}=-\frac{a}{b}$$

Example:
Exercise:

Problem: Simplify: $\frac{4(-3)+6(-2)}{-3(2)-2}$.

Solution:
Solution

The fraction bar acts like a grouping symbol. So completely simplify the numerator and the denominator separately.

	$\frac{4(-3)+6(-2)}{-3(2)-2}$
Multiply.	$\frac{-12+(-12)}{-6-2}$
Simplify.	$\frac{-24}{-8}$
Divide.	3

Note:
Exercise:

Problem: Simplify: $\frac{8(-2)+4(-3)}{-5(2)+3}$.

Solution:

4

Note:
Exercise:

Problem: Simplify: $\frac{7(-1)+9(-3)}{-5(3)-2}$.

Solution:

2

Translate Phrases to Expressions with Fractions

Now that we have done some work with fractions, we are ready to translate phrases that would result in expressions with fractions.

The English words quotient and ratio are often used to describe fractions. Remember that "quotient" means division. The quotient of a and b is the result we get from dividing a by b, or $\frac{a}{b}$.

Example:
Exercise:

Problem:

Translate the English phrase into an algebraic expression: the quotient of the difference of *m* and *n*, and *p*.

Solution:
Solution

We are looking for the *quotient of* the difference of *m* and *n*, *and p*. This means we want to divide the difference of m and n by p.
Equation:

$$\frac{m-n}{p}$$

Note:
Exercise:

Problem:

Translate the English phrase into an algebraic expression: the quotient of the difference of a and b, and cd.

Solution:

$\frac{a-b}{cd}$

Note:
Exercise:

Problem:

Translate the English phrase into an algebraic expression: the quotient of the sum of p and q, and r

Solution:

$\frac{p+q}{r}$

Key Concepts

- **Equivalent Fractions Property:** If a, b, c are numbers where $b \neq 0, c \neq 0$, then $\frac{a}{b} = \frac{a \cdot c}{b \cdot c}$ and $\frac{a \cdot c}{b \cdot c} = \frac{a}{b}$.

- **Fraction Division:** If a, b, c and d are numbers where $b \neq 0, c \neq 0,$ and $d \neq 0$, then $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$. To divide fractions, multiply the first fraction by the reciprocal of the second.
- **Fraction Multiplication:** If a, b, c and d are numbers where $b \neq 0,$ and $d \neq 0$, then $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$. To multiply fractions, multiply the numerators and multiply the denominators.
- **Placement of Negative Sign in a Fraction:** For any positive numbers a and b, $\frac{-a}{b} = \frac{a}{-b} = -\frac{a}{b}$.
- **Property of One:** $\frac{a}{a} = 1$; Any number, except zero, divided by itself is one.
- **Simplify a Fraction**

  Rewrite the numerator and denominator to show the common factors. If needed, factor the numerator and denominator into prime numbers first.
  Simplify using the equivalent fractions property by dividing out common factors.
  Multiply any remaining factors.

- **Simplify an Expression with a Fraction Bar**

  Simplify the expression in the numerator. Simplify the expression in the denominator.
  Simplify the fraction.

Practice Makes Perfect

Find Equivalent Fractions

In the following exercises, find three fractions equivalent to the given fraction. Show your work, using figures or algebra.

Exercise:

Problem: $\frac{3}{8}$

Solution:

$\frac{6}{16}, \frac{9}{24}, \frac{12}{32}$ answers may vary

Exercise:

Problem: $\frac{5}{8}$

Exercise:

Problem: $\frac{5}{9}$

Solution:

$\frac{10}{18}, \frac{15}{27}, \frac{20}{36}$ answers may vary

Exercise:

Problem: $\frac{1}{8}$

Simplify Fractions

In the following exercises, simplify.

Exercise:

Problem: $-\frac{40}{88}$

Solution:

$-\frac{5}{11}$

Exercise:

Problem: $-\frac{63}{99}$

Exercise:

Problem: $-\frac{108}{63}$

Solution:

$-\frac{12}{7}$

Exercise:

Problem: $-\frac{104}{48}$

Exercise:

Problem: $\frac{120}{252}$

Solution:

$\frac{10}{21}$

Exercise:

Problem: $\frac{182}{294}$

Exercise:

Problem: $-\frac{3x}{12y}$

Solution:

$-\frac{x}{4y}$

Exercise:

Problem: $-\frac{4x}{32y}$

Exercise:

Problem: $\frac{14x^2}{21y}$

Solution:

$\frac{2x^2}{3y}$

Exercise:

Problem: $\frac{24a}{32b^2}$

Multiply Fractions

In the following exercises, multiply.

Exercise:

Problem: $\frac{3}{4} \cdot \frac{9}{10}$

Solution:

$\frac{27}{40}$

Exercise:

Problem: $\frac{4}{5} \cdot \frac{2}{7}$

Exercise:

Problem: $-\frac{2}{3}\left(-\frac{3}{8}\right)$

Solution:

$\frac{1}{4}$

Exercise:

Problem: $-\frac{3}{4}\left(-\frac{4}{9}\right)$

Exercise:

Problem: $-\frac{5}{9}\cdot\frac{3}{10}$

Solution:

$-\frac{1}{6}$

Exercise:

Problem: $-\frac{3}{8}\cdot\frac{4}{15}$

Exercise:

Problem: $\left(-\frac{14}{15}\right)\left(\frac{9}{20}\right)$

Solution:

$-\frac{21}{50}$

Exercise:

Problem: $\left(-\frac{9}{10}\right)\left(\frac{25}{33}\right)$

Exercise:

Problem: $\left(-\frac{63}{84}\right)\left(-\frac{44}{90}\right)$

Solution:

$\frac{11}{30}$

Exercise:

Problem: $\left(-\frac{63}{60}\right)\left(-\frac{40}{88}\right)$

Exercise:

Problem: $4 \cdot \frac{5}{11}$

Solution:

$\frac{20}{11}$

Exercise:

Problem: $5 \cdot \frac{8}{3}$

Exercise:

Problem: $\frac{3}{7} \cdot 21n$

Solution:

$9n$

Exercise:

Problem: $\frac{5}{6} \cdot 30m$

Exercise:

Problem: $-8\left(\frac{17}{4}\right)$

Solution:

-34

Exercise:

Problem: $(-1)\left(-\frac{6}{7}\right)$

Divide Fractions

In the following exercises, divide.

Exercise:

Problem: $\frac{3}{4} \div \frac{2}{3}$

Solution:

$\frac{9}{8}$

Exercise:

Problem: $\frac{4}{5} \div \frac{3}{4}$

Exercise:

Problem: $-\frac{7}{9} \div \left(-\frac{7}{4}\right)$

Solution:

1

Exercise:

Problem: $-\frac{5}{6} \div \left(-\frac{5}{6}\right)$

Exercise:

Problem: $\frac{3}{4} \div \frac{x}{11}$

Solution:

$\frac{33}{4x}$

Exercise:

Problem: $\frac{2}{5} \div \frac{y}{9}$

Exercise:

Problem: $\frac{5}{18} \div \left(- \frac{15}{24}\right)$

Solution:

$- \frac{4}{9}$

Exercise:

Problem: $\frac{7}{18} \div \left(- \frac{14}{27}\right)$

Exercise:

Problem: $\frac{8u}{15} \div \frac{12v}{25}$

Solution:

$\frac{10u}{9v}$

Exercise:

Problem: $\frac{12r}{25} \div \frac{18s}{35}$

Exercise:

Problem: $-5 \div \frac{1}{2}$

Solution:

-10

Exercise:

Problem: $-3 \div \frac{1}{4}$

Exercise:

Problem: $\frac{3}{4} \div (-12)$

Solution:

$-\frac{1}{16}$

Exercise:

Problem: $-15 \div \left(-\frac{5}{3}\right)$

In the following exercises, simplify.

Exercise:

Problem: $\frac{-\frac{8}{21}}{\frac{12}{35}}$

Solution:

$-\frac{10}{9}$

Exercise:

Problem: $\frac{-\frac{9}{16}}{\frac{33}{40}}$

Exercise:

Problem: $\frac{-\frac{4}{5}}{2}$

Solution:

$-\frac{2}{5}$

Exercise:

Problem: $\frac{5}{\frac{3}{10}}$

Exercise:

Problem: $\frac{\frac{m}{3}}{\frac{n}{2}}$

Solution:

$\frac{2m}{3n}$

Exercise:

Problem: $\frac{-\frac{3}{8}}{-\frac{y}{12}}$

Simplify Expressions Written with a Fraction Bar

In the following exercises, simplify.

Exercise:

Problem: $\frac{22+3}{10}$

Solution:

$\frac{5}{2}$

Exercise:

Problem: $\frac{19-4}{6}$

Exercise:

Problem: $\frac{48}{24-15}$

Solution:

$\frac{16}{3}$

Exercise:

Problem: $\frac{46}{4+4}$

Exercise:

Problem: $\frac{-6+6}{8+4}$

Solution:

0

Exercise:

Problem: $\frac{-6+3}{17-8}$

Exercise:

Problem: $\frac{4\cdot3}{6\cdot6}$

Solution:

$\frac{1}{3}$

Exercise:

Problem: $\frac{6\cdot6}{9\cdot2}$

Exercise:

Problem: $\frac{4^2-1}{25}$

Solution:

$\frac{3}{5}$

Exercise:

Problem: $\frac{7^2+1}{60}$

Exercise:

Problem: $\frac{8\cdot3+2\cdot9}{14+3}$

Solution:

$2\frac{8}{17}$

Exercise:

Problem: $\frac{9\cdot6-4\cdot7}{22+3}$

Exercise:

Problem: $\frac{5\cdot6-3\cdot4}{4\cdot5-2\cdot3}$

Solution:

$\frac{3}{5}$

Exercise:

Problem: $\frac{8\cdot9-7\cdot6}{5\cdot6-9\cdot2}$

Exercise:

Problem: $\frac{5^2-3^2}{3-5}$

Solution:

$\frac{5}{2}$

Exercise:

Problem: $\frac{6^2-4^2}{4-6}$

Exercise:

Problem: $\frac{7\cdot4-2(8-5)}{9\cdot3-3\cdot5}$

Solution:

$\frac{11}{6}$

Exercise:

Problem: $\frac{9\cdot7-3(12-8)}{8\cdot7-6\cdot6}$

Exercise:

Problem: $\frac{9(8-2)-3(15-7)}{6(7-1)-3(17-9)}$

Solution:

$\frac{5}{2}$

Exercise:

Problem: $\frac{8(9-2)-4(14-9)}{7(8-3)-3(16-9)}$

Translate Phrases to Expressions with Fractions

In the following exercises, translate each English phrase into an algebraic expression.

Exercise:

Problem: the quotient of r and the sum of s and 10

Solution:

$\frac{r}{s+10}$

Exercise:

Problem: the quotient of A and the difference of 3 and B

Exercise:

Problem: the quotient of the difference of x and $y,$ and -3

Solution:

$\frac{x-y}{-8}$

Exercise:

Problem: the quotient of the sum of m and $n,$ and $4q$

Everyday Math

Exercise:

Problem:

Baking. A recipe for chocolate chip cookies calls for $\frac{3}{4}$ cup brown sugar. Imelda wants to double the recipe. ⓐ How much brown sugar will Imelda need? Show your calculation. ⓑ Measuring cups usually come in sets of $\frac{1}{4}, \frac{1}{3}, \frac{1}{2},$ and 1 cup. Draw a diagram to show two different ways that Imelda could measure the brown sugar needed to double the cookie recipe.

Solution:

ⓐ $1\frac{1}{2}$ cups ⓑ answers will vary

Exercise:

Problem:

Baking. Nina is making 4 pans of fudge to serve after a music recital. For each pan, she needs $\frac{2}{3}$ cup of condensed milk. ⓐ How much condensed milk will Nina need? Show your calculation. ⓑ Measuring cups usually come in sets of $\frac{1}{4}, \frac{1}{3}, \frac{1}{2},$ and 1 cup. Draw a diagram to show two different ways that Nina could measure the condensed milk needed for 4 pans of fudge.

Exercise:

Problem:

Portions Don purchased a bulk package of candy that weighs 5 pounds. He wants to sell the candy in little bags that hold $\frac{1}{4}$ pound. How many little bags of candy can he fill from the bulk package?

Solution:

20 bags

Exercise:

Problem:

Portions Kristen has $\frac{3}{4}$ yards of ribbon that she wants to cut into 6 equal parts to make hair ribbons for her daughter's 6 dolls. How long will each doll's hair ribbon be?

Writing Exercises

Exercise:

Problem:

Rafael wanted to order half a medium pizza at a restaurant. The waiter told him that a medium pizza could be cut into 6 or 8 slices. Would he prefer 3 out of 6 slices or 4 out of 8 slices? Rafael replied that since he wasn't very hungry, he would prefer 3 out of 6 slices. Explain what is wrong with Rafael's reasoning.

Solution:

Answers may vary

Exercise:

Problem:

Give an example from everyday life that demonstrates how $\frac{1}{2} \cdot \frac{2}{3}$ is $\frac{1}{3}$.

Exercise:

Problem: Explain how you find the reciprocal of a fraction.

Solution:

Answers may vary

Exercise:

Problem: Explain how you find the reciprocal of a negative number.

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
find equivalent fractions.			
simplify fractions.			
multiply fractions.			
divide fractions.			
simplify expressions written with a fraction bar.			
translate phrases to expressions with fractions.			

ⓑ After looking at the checklist, do you think you are well prepared for the next section? Why or why not?

Glossary

complex fraction
: A complex fraction is a fraction in which the numerator or the denominator contains a fraction.

denominator
: The denominator is the value on the bottom part of the fraction that indicates the number of equal parts into which the whole has been divided.

equivalent fractions
: Equivalent fractions are fractions that have the same value.

fraction

A fraction is written $\frac{a}{b}$, where $b \neq 0$ a is the numerator and b is the denominator. A fraction represents parts of a whole. The denominator b is the number of equal parts the whole has been divided into, and the numerator a indicates how many parts are included.

numerator

The numerator is the value on the top part of the fraction that indicates how many parts of the whole are included.

reciprocal

The reciprocal of $\frac{a}{b}$ is $\frac{b}{a}$. A number and its reciprocal multiply to one: $\frac{a}{b} \cdot \frac{b}{a} = 1$.

simplified fraction

A fraction is considered simplified if there are no common factors in its numerator and denominator.

Add and Subtract Fractions
By the end of this section, you will be able to:

- Add or subtract fractions with a common denominator
- Add or subtract fractions with different denominators
- Use the order of operations to simplify complex fractions
- Evaluate variable expressions with fractions

Note:
A more thorough introduction to the topics covered in this section can be found in the *Prealgebra* chapter, **Fractions**.

Add or Subtract Fractions with a Common Denominator

When we multiplied fractions, we just multiplied the numerators and multiplied the denominators right straight across. To add or subtract fractions, they must have a common denominator.

Note:
Fraction Addition and Subtraction
If $a, b,$ and c are numbers where $c \neq 0$, then
Equation:

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c} \quad \text{and} \quad \frac{a}{c} - \frac{b}{c} = \frac{a-b}{c}$$

To add or subtract fractions, add or subtract the numerators and place the result over the common denominator.

Note: Doing the Manipulative Mathematics activities “Model Fraction Addition” and “Model Fraction Subtraction” will help you develop a better understanding of adding and subtracting fractions.

Example:
Exercise:

Problem: Find the sum: $\frac{x}{3} + \frac{2}{3}$.

Solution:
Solution

$\frac{x}{3} + \frac{2}{3}$

Add the numerators and place the sum over the common denominator. $\frac{x+2}{3}$

Note:
Exercise:

Problem: Find the sum: $\frac{x}{4} + \frac{3}{4}$.

Solution:

$\frac{x+3}{4}$

Note:
Exercise:

Problem: Find the sum: $\frac{y}{8} + \frac{5}{8}$.

Solution:

$\frac{y+5}{8}$

Example:
Exercise:

Problem: Find the difference: $-\frac{23}{24} - \frac{13}{24}$.

Solution:
Solution

	$-\frac{23}{24}-\frac{13}{24}$
Subtract the numerators and place the difference over the common denominator.	$\frac{-23-13}{24}$
Simplify.	$\frac{-36}{24}$
Simplify. Remember, $-\frac{a}{b}=\frac{-a}{b}$.	$-\frac{3}{2}$

Note:
Exercise:

Problem: Find the difference: $-\frac{19}{28}-\frac{7}{28}$.

Solution:

$-\frac{26}{28}$

Note:
Exercise:

Problem: Find the difference: $-\frac{27}{32}-\frac{1}{32}$.

Solution:

$-\frac{7}{8}$

Example:
Exercise:

Problem: Simplify: $-\frac{10}{x}-\frac{4}{x}$.

Solution:
Solution

	$-\frac{10}{x} - \frac{4}{x}$
Subtract the numerators and place the difference over the common denominator.	$\frac{-14}{x}$
Rewrite with the sign in front of the fraction.	$-\frac{14}{x}$

Note:
Exercise:

Problem: Find the difference: $-\frac{9}{x} - \frac{7}{x}$.

Solution:

$-\frac{16}{x}$

Note:
Exercise:

Problem: Find the difference: $-\frac{17}{a} - \frac{5}{a}$.

Solution:

$-\frac{22}{a}$

Now we will do an example that has both addition and subtraction.

Example:
Exercise:

Problem: Simplify: $\frac{3}{8} + \left(-\frac{5}{8}\right) - \frac{1}{8}$.

Solution:
Solution

Add and subtract fractions—do they have a common denominator? Yes.	$\frac{3}{8} + \left(-\frac{5}{8}\right) - \frac{1}{8}$
Add and subtract the numerators and place the result over the common denominator.	$\frac{3+(-5)-1}{8}$
Simplify left to right.	$\frac{-2-1}{8}$
Simplify.	$-\frac{3}{8}$

Note:
Exercise:

Problem: Simplify: $\frac{2}{5} + \left(-\frac{4}{9}\right) - \frac{7}{9}$.

Solution:

-1

Note:
Exercise:

Problem: Simplify: $\frac{5}{9} + \left(-\frac{4}{9}\right) - \frac{7}{9}$.

Solution:

$-\frac{2}{3}$

Add or Subtract Fractions with Different Denominators

As we have seen, to add or subtract fractions, their denominators must be the same. The **least common denominator** (LCD) of two fractions is the smallest number that can be used as a common denominator of the fractions. The LCD of the two fractions is the least common multiple (LCM) of their denominators.

Note:
Least Common Denominator

The least common denominator (LCD) of two fractions is the least common multiple (LCM) of their denominators.

Note: Doing the Manipulative Mathematics activity “Finding the Least Common Denominator” will help you develop a better understanding of the LCD.

After we find the least common denominator of two fractions, we convert the fractions to equivalent fractions with the LCD. Putting these steps together allows us to add and subtract fractions because their denominators will be the same!

Example:
How to Add or Subtract Fractions
Exercise:

Problem: Add: $\frac{7}{12}+\frac{5}{18}$.

Solution:

Step 1. Do they have a common denominator?	No.	$12=2\cdot2\cdot3$ $18=2\cdot3\cdot3$
No—rewrite each fraction with the LCD (least common denominator).	Find the LCD of 12, 18.	$\text{LCD}=2\cdot2\cdot3\cdot3$ $\text{LCD}=36$
	Change into equivalent fractions with the LCD, 36.	$\frac{7}{12}+\frac{5}{18}$ $\frac{7\cdot3}{12\cdot3}+\frac{5\cdot2}{18\cdot2}$
	Do not simplify the equivalent fractions! If you do, you'll get back to the original fractions and lose the common denominator!	$\frac{21}{36}+\frac{10}{36}$
Step 2. Add or subtract the fractions.	Add.	$\frac{31}{36}$
Step 3. Simplify, if possible.	Because 31 is a prime number, it has no factors in common with 36. The answer is simplified.	

Note:
Exercise:

Problem: Add: $\frac{7}{12} + \frac{11}{15}$.

Solution:

$\frac{79}{60}$

Note:
Exercise:

Problem: Add: $\frac{13}{15} + \frac{17}{20}$.

Solution:

$\frac{103}{60}$

Note:
Add or Subtract Fractions.

Do they have a common denominator?

- Yes—go to step 2.
- No—rewrite each fraction with the LCD (least common denominator). Find the LCD. Change each fraction into an equivalent fraction with the LCD as its denominator.

Add or subtract the fractions.
Simplify, if possible.

When finding the equivalent fractions needed to create the common denominators, there is a quick way to find the number we need to multiply both the numerator and denominator. This method works if we found the LCD by factoring into primes.

Look at the factors of the LCD and then at each column above those factors. The “missing” factors of each denominator are the numbers we need.

missing factors

$$12 = 2 \cdot 2 \cdot 3$$
$$18 = 2 \cdot \quad 3 \cdot 3$$
$$\text{LCD} = 2 \cdot 2 \cdot 3 \cdot 3$$
$$\text{LCD} = 36$$

In [link], the LCD, 36, has two factors of 2 and two factors of 3.

The numerator 12 has two factors of 2 but only one of 3—so it is “missing” one 3—we multiply the numerator and denominator by 3.

The numerator 18 is missing one factor of 2—so we multiply the numerator and denominator by 2.

We will apply this method as we subtract the fractions in [link].

Example:
Exercise:

Problem: Subtract: $\frac{7}{15} - \frac{19}{24}$.

Solution:
Solution

Do the fractions have a common denominator? No, so we need to find the LCD.

Find the LCD. $\frac{7}{15} - \frac{19}{24}$ $15 = \quad 3 \cdot 5$ $24 = 2 \cdot 2 \cdot 2 \cdot 3$ $\text{LCD} = 2 \cdot 2 \cdot 2 \cdot 3 \cdot 5$ $\text{LCD} = 120$	
Notice, 15 is “missing” three factors of 2 and 24 is “missing” the 5 from the factors of the LCD. So we multiply 8 in the first fraction and 5 in the second fraction to get the LCD.	

Rewrite as equivalent fractions with the LCD.	$\frac{7 \cdot 8}{15 \cdot 8} - \frac{19 \cdot 5}{24 \cdot 5}$
Simplify.	$\frac{56}{120} - \frac{95}{120}$
Subtract.	$-\frac{39}{120}$
Check to see if the answer can be simplified.	$-\frac{13 \cdot 3}{40 \cdot 3}$
Both 39 and 120 have a factor of 3.	
Simplify.	$-\frac{13}{40}$

Do not simplify the equivalent fractions! If you do, you'll get back to the original fractions and lose the common denominator!

Note:
Exercise:

Problem: Subtract: $\frac{13}{24} - \frac{17}{32}$.

Solution:

$\frac{1}{96}$

Note:
Exercise:

Problem: Subtract: $\frac{21}{32} - \frac{9}{28}$.

Solution:

$\frac{75}{224}$

In the next example, one of the fractions has a variable in its numerator. Notice that we do the same steps as when both numerators are numbers.

Example:
Exercise:

Problem: Add: $\frac{3}{5}+\frac{x}{8}$.

Solution:
Solution

The fractions have different denominators.

		$\frac{3}{5}+\frac{x}{8}$
Find the LCD. $\begin{array}{r} 5 = 5 \\ 8 = 2\cdot 2\cdot 2 \\ \hline \text{LCD} = 2\cdot 2\cdot 2\cdot 5 \\ \text{LCD} = 40 \end{array}$		
Rewrite as equivalent fractions with the LCD.		$\frac{3\cdot 8}{5\cdot 8}+\frac{x\cdot 5}{8\cdot 5}$
Simplify.		$\frac{24}{40}+\frac{5x}{40}$
Add.		$\frac{24+5x}{40}$

Remember, we can only add like terms: 24 and 5*x* are not like terms.

Note:
Exercise:

Problem: Add: $\frac{y}{6}+\frac{7}{9}$.

Solution:

$\frac{9y+42}{54}$

Note:
Exercise:

Problem: Add: $\frac{x}{6}+\frac{7}{15}$.

Solution:

$\frac{15x+42}{135}$

We now have all four operations for fractions. [link] summarizes fraction operations.

Fraction Multiplication	**Fraction Division**
$\frac{a}{b}\cdot\frac{c}{d}=\frac{ac}{bd}$ Multiply the numerators and multiply the denominators	$\frac{a}{b}\div\frac{c}{d}=\frac{a}{b}\cdot\frac{d}{c}$ Multiply the first fraction by the reciprocal of the second.
Fraction Addition	**Fraction Subtraction**
$\frac{a}{c}+\frac{b}{c}=\frac{a+b}{c}$ Add the numerators and place the sum over the common denominator.	$\frac{a}{c}-\frac{b}{c}=\frac{a-b}{c}$ Subtract the numerators and place the difference over the common denominator.
To multiply or divide fractions, an LCD is NOT needed. To add or subtract fractions, an LCD is needed.	

Example:
Exercise:

Problem: Simplify: ⓐ $\frac{5x}{6} - \frac{3}{10}$ ⓑ $\frac{5x}{6} \cdot \frac{3}{10}$.

Solution:
Solution

First ask, "What is the operation?" Once we identify the operation that will determine whether we need a common denominator. Remember, we need a common denominator to add or subtract, but not to multiply or divide.

ⓐ What is the operation? The operation is subtraction.

Do the fractions have a common denominator? No. $\frac{5x}{6} - \frac{3}{10}$

Rewrite each fraction as an equivalent fraction with the LCD. $\frac{5x\cdot 5}{6\cdot 5} - \frac{3\cdot 3}{10\cdot 3}$

$\frac{25x}{30} - \frac{9}{30}$

Subtract the numerators and place the difference over the common denominators. $\frac{25x-9}{30}$

Simplify, if possible There are no common factors.

The fraction is simplified.

ⓑ What is the operation? Multiplication.

$\frac{5x}{6} \cdot \frac{3}{10}$

To multiply fractions, multiply the numerators and multiply the denominators. $\frac{5x\cdot 3}{6\cdot 10}$

Rewrite, showing common factors.

Remove common factors. $\frac{\cancel{5}\,x\cdot\cancel{3}}{2\cdot\cancel{3}\cdot 2\cdot\cancel{5}}$

Simplify. $\frac{x}{4}$

Notice we needed an LCD to add $\frac{5x}{6} - \frac{3}{10}$, but not to multiply $\frac{5x}{6} \cdot \frac{3}{10}$.

Note:
Exercise:

Problem: Simplify: ⓐ $\frac{3a}{4} - \frac{8}{9}$ ⓑ $\frac{3a}{4} \cdot \frac{8}{9}$.

Solution:

ⓐ $\frac{27a-32}{36}$ ⓑ $\frac{2a}{3}$

Note:
Exercise:

Problem: Simplify: ⓐ $\frac{4k}{5} - \frac{1}{6}$ ⓑ $\frac{4k}{5} \cdot \frac{1}{6}$.

Solution:

ⓐ $\frac{24k-5}{30}$ ⓑ $\frac{2k}{15}$

Use the Order of Operations to Simplify Complex Fractions

We have seen that a complex fraction is a fraction in which the numerator or denominator contains a fraction. The fraction bar indicates division. We simplified the complex fraction $\frac{\frac{3}{4}}{\frac{5}{8}}$ by dividing $\frac{3}{4}$ by $\frac{5}{8}$.

Now we'll look at complex fractions where the numerator or denominator contains an expression that can be simplified. So we first must completely simplify the numerator and denominator separately using the order of operations. Then we divide the numerator by the denominator.

Example:
How to Simplify Complex Fractions
Exercise:

Problem: Simplify: $\frac{\left(\frac{1}{2}\right)^2}{4+3^2}$.

Solution:
Solution

Step 1. Simplify the numerator. * Remember, $\left(\frac{1}{2}\right)^2$ means $\frac{1}{2}\cdot\frac{1}{2}$.	$\dfrac{\left(\frac{1}{2}\right)^2}{4+3^2}$ $\dfrac{\frac{1}{4}}{4+3^2}$
Step 2. Simplify the denominator.	$\dfrac{\frac{1}{4}}{4+9}$ $\dfrac{\frac{1}{4}}{13}$
Step 3. Divide the numerator by the denominator. Simplify if possible. * Remember, $13=\frac{13}{1}$	$\frac{1}{4}\div 13$ $\frac{1}{4}\cdot\frac{1}{13}$ $\frac{1}{52}$

Note:
Exercise:

Problem: Simplify: $\frac{\left(\frac{1}{3}\right)^2}{2^3+2}$.

Solution:

$\frac{1}{90}$

Note:
Exercise:

Problem: Simplify: $\frac{1+4^2}{\left(\frac{1}{4}\right)^2}$.

Solution:

272

Note:
Simplify Complex Fractions.

Simplify the numerator.
Simplify the denominator.
Divide the numerator by the denominator. Simplify if possible.

Example:
Exercise:

Problem: Simplify: $\frac{\frac{1}{2}+\frac{2}{3}}{\frac{3}{4}-\frac{1}{6}}$.

Solution:
Solution

It may help to put parentheses around the numerator and the denominator.

	$\frac{\left(\frac{1}{2}+\frac{2}{3}\right)}{\left(\frac{3}{4}-\frac{1}{6}\right)}$
Simplify the numerator $(\text{LCD} = 6)$ and simplify the denominator $(\text{LCD} = 12)$.	$\frac{\left(\frac{3}{6}+\frac{4}{6}\right)}{\left(\frac{9}{12}-\frac{2}{12}\right)}$
Simplify.	$\frac{\left(\frac{7}{6}\right)}{\left(\frac{7}{12}\right)}$
Divide the numerator by the denominator.	$\frac{7}{6} \div \frac{7}{12}$
Simplify.	$\frac{7}{6} \cdot \frac{12}{7}$
Divide out common factors.	$\frac{7 \cdot 6 \cdot 2}{6 \cdot 7}$
Simplify.	2

Note:
Exercise:

Problem: Simplify: $\frac{\frac{1}{3}+\frac{1}{2}}{\frac{3}{4}-\frac{1}{3}}$.

Solution:

2

Note:
Exercise:

Problem: Simplify: $\frac{\frac{2}{3}-\frac{1}{2}}{\frac{1}{4}+\frac{1}{3}}$.

Solution:

$\frac{2}{7}$

Evaluate Variable Expressions with Fractions

We have evaluated expressions before, but now we can evaluate expressions with fractions. Remember, to evaluate an expression, we substitute the value of the variable into the expression and then simplify.

Example:
Exercise:

Problem: Evaluate $x+\frac{1}{3}$ when ⓐ $x=-\frac{1}{3}$ ⓑ $x=-\frac{3}{4}$.

Solution:

ⓐ To evaluate $x+\frac{1}{3}$ when $x=-\frac{1}{3}$, substitute $-\frac{1}{3}$ for x in the expression.

	$x+\frac{1}{3}$

Substitute $-\frac{1}{3}$ for x.	$-\frac{1}{3}+\frac{1}{3}$
Simplify.	0

ⓑ To evaluate $x+\frac{1}{3}$ when $x=-\frac{3}{4}$, we substitute $-\frac{3}{4}$ for x in the expression.

	$x+\frac{1}{3}$
Substitute $-\frac{3}{4}$ for x.	$-\frac{3}{4}+\frac{1}{3}$
Rewrite as equivalent fractions with the LCD, 12.	$-\frac{3\cdot 3}{4\cdot 3}+\frac{1\cdot 4}{3\cdot 4}$
Simplify.	$-\frac{9}{12}+\frac{4}{12}$
Add.	$-\frac{5}{12}$

Note:
Exercise:

Problem: Evaluate $x+\frac{3}{4}$ when ⓐ $x=-\frac{7}{4}$ ⓑ $x=-\frac{5}{4}$.

Solution:

ⓐ -1 ⓑ $-\frac{1}{2}$

Note:
Exercise:

Problem: Evaluate $y+\frac{1}{2}$ when ⓐ $y=\frac{2}{3}$ ⓑ $y=-\frac{3}{4}$.

Solution:

ⓐ $\frac{7}{6}$ ⓑ $-\frac{1}{12}$

Example:
Exercise:

Problem: Evaluate $-\frac{5}{6}-y$ when $y=-\frac{2}{3}$.

Solution:
Solution

	$-\frac{5}{6}-y$
Substitute $-\frac{2}{3}$ for y.	$-\frac{5}{6}-\left(-\frac{2}{3}\right)$
Rewrite as equivalent fractions with the LCD, 6.	$-\frac{5}{6}-\left(-\frac{4}{6}\right)$
Subtract.	$\frac{-5-(-4)}{6}$
Simplify.	$-\frac{1}{6}$

Note:
Exercise:

Problem: Evaluate $-\frac{1}{2}-y$ when $y=-\frac{1}{4}$.

Solution:

$-\frac{1}{4}$

Note:
Exercise:

Problem: Evaluate $-\frac{3}{8}-y$ when $x=-\frac{5}{2}$.

Solution:

$-\frac{17}{8}$

Example:
Exercise:

Problem: Evaluate $2x^2y$ when $x=\frac{1}{4}$ and $y=-\frac{2}{3}$.

Solution:
Solution

Substitute the values into the expression.

	$2x^2y$
Substitute $\frac{1}{4}$ for x and $-\frac{2}{3}$ for y.	$2\left(\frac{1}{4}\right)^2\left(-\frac{2}{3}\right)$
Simplify exponents first.	$2\left(\frac{1}{16}\right)\left(-\frac{2}{3}\right)$

Multiply. Divide out the common factors. Notice we write 16 as $2 \cdot 2 \cdot 4$ to make it easy to remove common factors.	$-\frac{\cancel{2} \cdot 1 \cdot \cancel{2}}{\cancel{2} \cdot \cancel{2} \cdot 4 \cdot 3}$
Simplify.	$-\frac{1}{12}$

Note:
Exercise:

Problem: Evaluate $3ab^2$ when $a = -\frac{2}{3}$ and $b = -\frac{1}{2}$.

Solution:

$-\frac{1}{2}$

Note:
Exercise:

Problem: Evaluate $4c^3d$ when $c = -\frac{1}{2}$ and $d = -\frac{4}{3}$.

Solution:

$\frac{2}{3}$

The next example will have only variables, no constants.

Example:
Exercise:

Problem: Evaluate $\frac{p+q}{r}$ when $p = -4, q = -2,$ and $r = 8$.

Solution:
Solution

To evaluate $\frac{p+q}{r}$ when $p = -4, q = -2,$ and $r = 8$, we substitute the values into the expression.

	$\frac{p+q}{r}$
Substitute −4 for *p*, −2 for *q* and 8 for *r*.	$\frac{-4+(-2)}{8}$
Add in the numerator first.	$\frac{-6}{8}$
Simplify.	$-\frac{3}{4}$

Note:
Exercise:

Problem: Evaluate $\frac{a+b}{c}$ when $a=-8, b=-7,$ and $c=6.$

Solution:

$-\frac{5}{2}$

Note:
Exercise:

Problem: Evaluate $\frac{x+y}{z}$ when $x=9, y=-18,$ and $z=-6.$

Solution:

$\frac{3}{2}$

Key Concepts

- **Fraction Addition and Subtraction:** If $a, b,$ and c are numbers where $c \neq 0$, then $\frac{a}{c}+\frac{b}{c}=\frac{a+b}{c}$ and $\frac{a}{c}-\frac{b}{c}=\frac{a-b}{c}$.
  To add or subtract fractions, add or subtract the numerators and place the result over the common denominator.
- **Strategy for Adding or Subtracting Fractions**

Do they have a common denominator?	Yes—go to step 2.	No—Rewrite each fraction with the LCD (Least Common Denominator). Find the LCD. Change each fraction into an equivalent fraction with the LCD as its denominator.

Add or subtract the fractions.
Simplify, if possible. To multiply or divide fractions, an LCD IS NOT needed. To add or subtract fractions, an LCD IS needed.

- **Simplify Complex Fractions**

  Simplify the numerator.
  Simplify the denominator.
  Divide the numerator by the denominator. Simplify if possible.

Practice Makes Perfect

Add and Subtract Fractions with a Common Denominator

In the following exercises, add.

Exercise:

Problem: $\frac{6}{13}+\frac{5}{13}$

Solution:

$\frac{11}{13}$

Exercise:

Problem: $\frac{4}{15}+\frac{7}{15}$

Exercise:

Problem: $\frac{x}{4}+\frac{3}{4}$

Solution:

$\frac{x+3}{4}$

Exercise:

Problem: $\frac{8}{q}+\frac{6}{q}$

Exercise:

Problem: $-\frac{3}{16}+\left(-\frac{7}{16}\right)$

Solution:

$-\frac{5}{8}$

Exercise:

Problem: $-\frac{5}{16}+\left(-\frac{9}{16}\right)$

Exercise:

Problem: $-\frac{8}{17}+\frac{15}{17}$

Solution:

$\frac{7}{17}$

Exercise:

Problem: $-\frac{9}{19}+\frac{17}{19}$

Exercise:

Problem: $\frac{6}{13}+\left(-\frac{10}{13}\right)+\left(-\frac{12}{13}\right)$

Solution:

$-\frac{16}{13}$

Exercise:

Problem: $\frac{5}{12}+\left(-\frac{7}{12}\right)+\left(-\frac{11}{12}\right)$

In the following exercises, subtract.

Exercise:

Problem: $\frac{11}{15}-\frac{7}{15}$

Solution:

$\frac{4}{15}$

Exercise:

Problem: $\frac{9}{13}-\frac{4}{13}$

Exercise:

Problem: $\frac{11}{12} - \frac{5}{12}$

Solution:

$\frac{1}{2}$

Exercise:

Problem: $\frac{7}{12} - \frac{5}{12}$

Exercise:

Problem: $\frac{19}{21} - \frac{4}{21}$

Solution:

$\frac{5}{7}$

Exercise:

Problem: $\frac{17}{21} - \frac{8}{21}$

Exercise:

Problem: $\frac{5y}{8} - \frac{7}{8}$

Solution:

$\frac{5y-7}{8}$

Exercise:

Problem: $\frac{11z}{13} - \frac{8}{13}$

Exercise:

Problem: $-\frac{23}{u} - \frac{15}{u}$

Solution:

$-\frac{38}{u}$

Exercise:

Problem: $-\frac{29}{v} - \frac{26}{v}$

Exercise:

Problem: $-\frac{3}{5}-\left(-\frac{4}{5}\right)$

Solution:

$\frac{1}{5}$

Exercise:

Problem: $-\frac{3}{7}-\left(-\frac{5}{7}\right)$

Exercise:

Problem: $-\frac{7}{9}-\left(-\frac{5}{9}\right)$

Solution:

$-\frac{2}{9}$

Exercise:

Problem: $-\frac{8}{11}-\left(-\frac{5}{11}\right)$

Mixed Practice

In the following exercises, simplify.

Exercise:

Problem: $-\frac{5}{18}\cdot\frac{9}{10}$

Solution:

$-\frac{1}{4}$

Exercise:

Problem: $-\frac{3}{14}\cdot\frac{7}{12}$

Exercise:

Problem: $\frac{n}{5}-\frac{4}{5}$

Solution:

$\frac{n-4}{5}$

Exercise:

Problem: $\frac{6}{11} - \frac{s}{11}$

Exercise:

Problem: $-\frac{7}{24} + \frac{2}{24}$

Solution:

$-\frac{5}{24}$

Exercise:

Problem: $-\frac{5}{18} + \frac{1}{18}$

Exercise:

Problem: $\frac{8}{15} \div \frac{12}{5}$

Solution:

$\frac{2}{9}$

Exercise:

Problem: $\frac{7}{12} \div \frac{9}{28}$

Add or Subtract Fractions with Different Denominators

In the following exercises, add or subtract.

Exercise:

Problem: $\frac{1}{2} + \frac{1}{7}$

Solution:

$\frac{9}{14}$

Exercise:

Problem: $\frac{1}{3} + \frac{1}{8}$

Exercise:

Problem: $\frac{1}{3} - \left(-\frac{1}{9}\right)$

Solution:

$\frac{4}{9}$

Exercise:

Problem: $\frac{1}{4} - \left(- \frac{1}{8}\right)$

Exercise:

Problem: $\frac{7}{12} + \frac{5}{8}$

Solution:

$\frac{29}{24}$

Exercise:

Problem: $\frac{5}{12} + \frac{3}{8}$

Exercise:

Problem: $\frac{7}{12} - \frac{9}{16}$

Solution:

$\frac{1}{48}$

Exercise:

Problem: $\frac{7}{16} - \frac{5}{12}$

Exercise:

Problem: $\frac{2}{3} - \frac{3}{8}$

Solution:

$\frac{7}{24}$

Exercise:

Problem: $\frac{5}{6} - \frac{3}{4}$

Exercise:

Problem: $- \frac{11}{30} + \frac{27}{40}$

Solution:

$\frac{37}{120}$

Exercise:

Problem: $-\frac{9}{20}+\frac{17}{30}$

Exercise:

Problem: $-\frac{13}{30}+\frac{25}{42}$

Solution:

$\frac{17}{105}$

Exercise:

Problem: $-\frac{23}{30}+\frac{5}{48}$

Exercise:

Problem: $-\frac{39}{56}-\frac{22}{35}$

Solution:

$-\frac{53}{40}$

Exercise:

Problem: $-\frac{33}{49}-\frac{18}{35}$

Exercise:

Problem: $-\frac{2}{3}-\left(-\frac{3}{4}\right)$

Solution:

$\frac{1}{12}$

Exercise:

Problem: $-\frac{3}{4}-\left(-\frac{4}{5}\right)$

Exercise:

Problem: $1 + \frac{7}{8}$

Solution:

$\frac{15}{8}$

Exercise:

Problem: $1 - \frac{3}{10}$

Exercise:

Problem: $\frac{x}{3} + \frac{1}{4}$

Solution:

$\frac{4x+3}{12}$

Exercise:

Problem: $\frac{y}{2} + \frac{2}{3}$

Exercise:

Problem: $\frac{y}{4} - \frac{3}{5}$

Solution:

$\frac{4y-12}{20}$

Exercise:

Problem: $\frac{x}{5} - \frac{1}{4}$

Mixed Practice

In the following exercises, simplify.

Exercise:

Problem: ⓐ $\frac{2}{3} + \frac{1}{6}$ ⓑ $\frac{2}{3} \div \frac{1}{6}$

Solution:

ⓐ $\frac{5}{6}$ ⓑ 4

Exercise:

Problem: ⓐ $-\frac{2}{5} - \frac{1}{8}$ ⓑ $-\frac{2}{5} \cdot \frac{1}{8}$

Exercise:

Problem: ⓐ $\frac{5n}{6} \div \frac{8}{15}$ ⓑ $\frac{5n}{6} - \frac{8}{15}$

Solution:

ⓐ $\frac{25n}{16}$ ⓑ $\frac{25n-16}{30}$

Exercise:

Problem: ⓐ $\frac{3a}{8} \div \frac{7}{12}$ ⓑ $\frac{3a}{8} - \frac{7}{12}$

Exercise:

Problem: $-\frac{3}{8} \div \left(-\frac{3}{10}\right)$

Solution:

$\frac{5}{4}$

Exercise:

Problem: $-\frac{5}{12} \div \left(-\frac{5}{9}\right)$

Exercise:

Problem: $-\frac{3}{8} + \frac{5}{12}$

Solution:

$\frac{1}{24}$

Exercise:

Problem: $-\frac{1}{8} + \frac{7}{12}$

Exercise:

Problem: $\frac{5}{6} - \frac{1}{9}$

Solution:

$\frac{13}{18}$

Exercise:

Problem: $\frac{5}{9} - \frac{1}{6}$

Exercise:

Problem: $-\frac{7}{15} - \frac{y}{4}$

Solution:

$\frac{-28-15y}{60}$

Exercise:

Problem: $-\frac{3}{8} - \frac{x}{11}$

Exercise:

Problem: $\frac{11}{12a} \cdot \frac{9a}{16}$

Solution:

$\frac{33}{64}$

Exercise:

Problem: $\frac{10y}{13} \cdot \frac{8}{15y}$

Use the Order of Operations to Simplify Complex Fractions

In the following exercises, simplify.

Exercise:

Problem: $\frac{2^3+4^2}{\left(\frac{2}{3}\right)^2}$

Solution:

54

Exercise:

Problem: $\frac{3^3-3^2}{\left(\frac{3}{4}\right)^2}$

Exercise:

Problem: $\frac{\left(\frac{3}{5}\right)^2}{\left(\frac{3}{7}\right)^2}$

Solution:

$\frac{49}{25}$

Exercise:

Problem: $\frac{\left(\frac{3}{4}\right)^2}{\left(\frac{5}{8}\right)^2}$

Exercise:

Problem: $\frac{2}{\frac{1}{3}+\frac{1}{5}}$

Solution:

$\frac{15}{4}$

Exercise:

Problem: $\frac{5}{\frac{1}{4}+\frac{1}{3}}$

Exercise:

Problem: $\frac{\frac{7}{8}-\frac{2}{3}}{\frac{1}{2}+\frac{3}{8}}$

Solution:

$\frac{5}{21}$

Exercise:

Problem: $\frac{\frac{3}{4}-\frac{3}{5}}{\frac{1}{4}+\frac{2}{5}}$

Exercise:

Problem: $\frac{1}{2}+\frac{2}{3}\cdot\frac{5}{12}$

Solution:

$\frac{7}{9}$

Exercise:

Problem: $\frac{1}{3}+\frac{2}{5}\cdot\frac{3}{4}$

Exercise:

Problem: $1-\ \frac{3}{5}\div\frac{1}{10}$

Solution:

-5

Exercise:

Problem: $1-\ \frac{5}{6}\div\frac{1}{12}$

Exercise:

Problem: $\frac{2}{3}+\frac{1}{6}+\frac{3}{4}$

Solution:

$\frac{19}{12}$

Exercise:

Problem: $\frac{2}{3}+\frac{1}{4}+\frac{3}{5}$

Exercise:

Problem: $\frac{3}{8}-\ \frac{1}{6}+\frac{3}{4}$

Solution:

$\frac{23}{24}$

Exercise:

Problem: $\frac{2}{5}+\frac{5}{8}-\ \frac{3}{4}$

Exercise:

Problem: $12\left(\frac{9}{20}-\ \frac{4}{15}\right)$

Solution:

$\frac{11}{5}$

Exercise:

Problem: $8\left(\frac{15}{16} - \frac{5}{6}\right)$

Exercise:

Problem: $\frac{\frac{5}{8}+\frac{1}{6}}{\frac{19}{24}}$

Solution:

1

Exercise:

Problem: $\frac{\frac{1}{6}+\frac{3}{10}}{\frac{14}{30}}$

Exercise:

Problem: $\left(\frac{5}{9} + \frac{1}{6}\right) \div \left(\frac{2}{3} - \frac{1}{2}\right)$

Solution:

$\frac{13}{3}$

Exercise:

Problem: $\left(\frac{3}{4} + \frac{1}{6}\right) \div \left(\frac{5}{8} - \frac{1}{3}\right)$

Evaluate Variable Expressions with Fractions

In the following exercises, evaluate.

Exercise:

Problem: $x + \left(-\frac{5}{6}\right)$ when
ⓐ $x = \frac{1}{3}$
ⓑ $x = -\frac{1}{6}$

Solution:

ⓐ $-\frac{1}{2}$ ⓑ -1

Exercise:

Problem: $x + \left(-\frac{11}{12}\right)$ when
ⓐ $x = \frac{11}{12}$
ⓑ $x = \frac{3}{4}$

Exercise:

Problem: $x - \frac{2}{5}$ when
ⓐ $x = \frac{3}{5}$
ⓑ $x = -\frac{3}{5}$

Solution:

ⓐ $\frac{1}{5}$ ⓑ -1

Exercise:

Problem: $x - \frac{1}{3}$ when
ⓐ $x = \frac{2}{3}$
ⓑ $x = -\frac{2}{3}$

Exercise:

Problem: $\frac{7}{10} - w$ when
ⓐ $w = \frac{1}{2}$
ⓑ $w = -\frac{1}{2}$

Solution:

ⓐ $\frac{1}{5}$ ⓑ $\frac{6}{5}$

Exercise:

Problem: $\frac{5}{12} - w$ when
ⓐ $w = \frac{1}{4}$
ⓑ $w = -\frac{1}{4}$

Exercise:

Problem: $2x^2y^3$ when $x = -\frac{2}{3}$ and $y = -\frac{1}{2}$

Solution:

$-\frac{1}{9}$

Exercise:

Problem: $8u^2v^3$ when $u = -\frac{3}{4}$ and $v = -\frac{1}{2}$

Exercise:

Problem: $\frac{a+b}{a-b}$ when $a = -3, b = 8$

Solution:

$-\frac{5}{11}$

Exercise:

Problem: $\frac{r-s}{r+s}$ when $r = 10, s = -5$

Everyday Math

Exercise:

Problem:

Decorating Laronda is making covers for the throw pillows on her sofa. For each pillow cover, she needs $\frac{1}{2}$ yard of print fabric and $\frac{3}{8}$ yard of solid fabric. What is the total amount of fabric Laronda needs for each pillow cover?

Solution:

$\frac{7}{8}$ yard

Exercise:

Problem:

Baking Vanessa is baking chocolate chip cookies and oatmeal cookies. She needs $\frac{1}{2}$ cup of sugar for the chocolate chip cookies and $\frac{1}{4}$ of sugar for the oatmeal cookies. How much sugar does she need altogether?

Writing Exercises

Exercise:

Problem: Why do you need a common denominator to add or subtract fractions? Explain.

Solution:

Answers may vary

Exercise:

Problem: How do you find the LCD of 2 fractions?

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can…	Confidently	With some help	No-I don't get it!
add and subtract fractions with different denominators.			
identify and use fraction operations.			
use the order of operations to simplify complex fractions.			
evaluate variable expressions with fractions.			

ⓑ After looking at the checklist, do you think you are well-prepared for the next chapter? Why or why not?

Glossary

least common denominator
: The least common denominator (LCD) of two fractions is the Least common multiple (LCM) of their denominators.

Decimals
By the end of this section, you will be able to:

- Name and write decimals
- Round decimals
- Add and subtract decimals
- Multiply and divide decimals
- Convert decimals, fractions, and percents

Note:
A more thorough introduction to the topics covered in this section can be found in the *Prealgebra* chapter, **Decimals**.

Name and Write Decimals

Decimals are another way of writing fractions whose denominators are powers of 10.
Equation:

$$\begin{array}{rclc} 0.1 & = & \frac{1}{10} & \qquad 0.1 \text{ is "one tenth"} \\ 0.01 & = & \frac{1}{100} & \qquad 0.01 \text{ is "one hundredth"} \\ 0.001 & = & \frac{1}{1{,}000} & \qquad 0.001 \text{ is "one thousandth"} \\ 0.0001 & = & \frac{1}{10{,}000} & \qquad 0.0001 \text{ is "one ten-thousandth"} \end{array}$$

Notice that “ten thousand” is a number larger than one, but “one ten-thousand**th**” is a number smaller than one. The “th” at the end of the name tells you that the number is smaller than one.

When we name a whole number, the name corresponds to the place value based on the powers of ten. We read 10,000 as “ten thousand” and 10,000,000 as “ten million.” Likewise, the names of the decimal places correspond to their fraction values. [link] shows the names of the place values to the left and right of the decimal point.

Place Value

Hundred thousands	Ten thousands	Thousands	Hundreds	Tens	Ones		Tenths	Hundredths	Thousandths	Ten-thousandths	Hundred-thousandths
						•					

Place value of decimal numbers are shown to the left and right of the decimal point.

Example:
How to Name Decimals
Exercise:

Problem: Name the decimal 4.3.

Solution:
Solution

Step 1. Name the number to the left of the decimal point.	4 is to the left of the decimal point.	4.3 four ________________
Step 2. Write 'and' for the decimal point.		four and______________
Step 3. Name the 'number' part to the right of the decimal point as if it were a whole number.	3 is to the right of the decimal point.	four and three _________
Step 4. Name the decimal place.		four and three tenths

Note:
Exercise:

Problem: Name the decimal: 6.7.

Solution:

six and seven tenths

Note:
Exercise:

Problem: Name the decimal: 5.8.

Solution:

five and eight tenths

We summarize the steps needed to name a decimal below.

Note:
Name a Decimal.

Name the number to the left of the decimal point.

Write “and” for the decimal point.
Name the “number” part to the right of the decimal point as if it were a whole number.
Name the decimal place of the last digit.

Example:
Exercise:

Problem: Name the decimal: -15.571.

Solution:
Solution

	-15.571
Name the number to the left of the decimal point.	negative fifteen ______________________
Write “and” for the decimal point.	negative fifteen and ____________________
Name the number to the right of the decimal point.	negative fifteen and five hundred seventy-on
The 1 is in the thousandths place.	negative fifteen and five hundred seventy-on

Note:
Exercise:

Problem: Name the decimal: -13.461.

Solution:

negative thirteen and four hundred sixty-one thousandths

Note:
Exercise:

Problem: Name the decimal: -2.053.

Solution:

negative two and fifty-three thousandths

When we write a check we write both the numerals and the name of the number. Let’s see how to write the decimal from the name.

Example:
How to Write Decimals
Exercise:

Problem: Write “fourteen and twenty-four thousandths” as a decimal.

Solution:
Solution

Step 1. Look for the word ‘and’; it locates the decimal point. Place a decimal point under the word ‘and’. Translate the words before ‘and’ into the whole number and place to the left of the decimal point.		fourteen and twenty-four thousandths fourteen and twenty-four thousandths ___ . ______ 14 . ______
Step 2. Mark the number of decimal places needed to the right of the decimal point by noting the place value indicated by the last word.	The last word is ‘thousandths’.	14 . ___ ___ ___ tenths hundredths thousandths
Step 3. Translate the words after ‘and’ into the number to the right of the decimal point. Write the number in the spaces – putting the final digit in the last place.		14 . ___ 2 4
Step 4. Fill in zeros for empty place holders as needed.	Zeros are needed in the tenths place.	14 . 0 2 4 Fourteen and twenty-four thousandths is written 14.024.

Note:
Exercise:

Problem: Write as a decimal: thirteen and sixty-eight thousandths.

Solution:

13.68

Note:
Exercise:

Problem: Write as a decimal: five and ninety-four thousandths.

Solution:

5.94

We summarize the steps to writing a decimal.

Note:
Write a decimal.

Look for the word “and”—it locates the decimal point.

- Place a decimal point under the word “and.” Translate the words before “and” into the whole number and place it to the left of the decimal point.
- If there is no “and,” write a “0” with a decimal point to its right.

Mark the number of decimal places needed to the right of the decimal point by noting the place value indicated by the last word.
Translate the words after “and” into the number to the right of the decimal point. Write the number in the spaces —putting the final digit in the last place.
Fill in zeros for place holders as needed.

Round Decimals

Rounding decimals is very much like rounding whole numbers. We will round decimals with a method based on the one we used to round whole numbers.

Example:
How to Round Decimals
Exercise:

Problem: Round 18.379 to the nearest hundredth.

Solution:

Step 1. Locate the given place value and mark it with an arrow.		hundredths place ↓ 18.379
Step 2. Underline the digit to the right of the given place value.		hundredths place ↓ 18.379
Step 3. Is this digit greater than or equal to 5? **Yes:** Add 1 to the digit in the given place value. **No:** Do not change the digit in the given place value.	Because 9 is greater than or equal to 5, add 1 to the 7.	18.37 9 add 1 delete

Step 4. Rewrite the number, removing all digits to the right of the rounding digit.		18.38 18.38 is 18.379 rounded to the nearest hundredth.

Note:
Exercise:

Problem: Round to the nearest hundredth: 1.047.

Solution:

1.05

Note:
Exercise:

Problem: Round to the nearest hundredth: 9.173.

Solution:

9.17

We summarize the steps for rounding a decimal here.

Note:
Round Decimals.

Locate the given place value and mark it with an arrow.
Underline the digit to the right of the place value.
Is this digit greater than or equal to 5?

- Yes—add 1 to the digit in the given place value.
- No—do not change the digit in the given place value.

Rewrite the number, deleting all digits to the right of the rounding digit.

Example:
Exercise:

Problem: Round 18.379 to the nearest ⓐ tenth ⓑ whole number.

Solution:
Solution

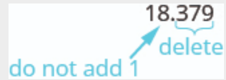
Round 18.379

ⓐ to the nearest tenth

Locate the tenths place with an arrow.	tenths place ↓ 18.379
Underline the digit to the right of the given place value.	tenths place ↓ 18.3<u>7</u>9
Because 7 is greater than or equal to 5, add 1 to the 3.	18.3$\underbrace{79}_{\text{delete}}$ add 1 ↗
Rewrite the number, deleting all digits to the right of the rounding digit.	18.4
Notice that the deleted digits were NOT replaced with zeros.	So, 18.379 rounded to the nearest tenth is 18.4.

ⓑ to the nearest whole number

Locate the ones place with an arrow.	ones place ↓ 18.379
Underline the digit to the right of the given place value.	ones place ↓ 18.<u>3</u>79

Since 3 is not greater than or equal to 5, do not add 1 to the 8.	
Rewrite the number, deleting all digits to the right of the rounding digit.	18
	So, 18.379 rounded to the nearest whole number is 18.

Note:
Exercise:

Problem: Round 6.582 to the nearest ⓐ hundredth ⓑ tenth ⓒ whole number.

Solution:

ⓐ 6.58 ⓑ 6.6 ⓒ 7

Note:
Exercise:

Problem: Round 15.2175 to the nearest ⓐ thousandth ⓑ hundredth ⓒ tenth.

Solution:

ⓐ 15.218 ⓑ 15.22 ⓒ 15.2

Add and Subtract Decimals

To add or subtract decimals, we line up the decimal points. By lining up the decimal points this way, we can add or subtract the corresponding place values. We then add or subtract the numbers as if they were whole numbers and then place the decimal point in the sum.

Note:
Add or Subtract Decimals.

Write the numbers so the decimal points line up vertically.
Use zeros as place holders, as needed.
Add or subtract the numbers as if they were whole numbers. Then place the decimal point in the answer under the decimal points in the given numbers.

Example:
Exercise:

Problem: Add: $23.5 + 41.38$.

Solution:
Solution

Write the numbers so the decimal points line up vertically.	$\begin{array}{r} 23.5 \\ +41.38 \\ \hline \end{array}$
Put 0 as a placeholder after the 5 in 23.5. Remember, $\frac{5}{10} = \frac{50}{100}$ so $0.5 = 0.50$.	$\begin{array}{r} 23.50 \\ +41.38 \\ \hline \end{array}$
Add the numbers as if they were whole numbers. Then place the decimal point in the sum.	$\begin{array}{r} 23.50 \\ +41.38 \\ \hline 64.88 \end{array}$

Note:
Exercise:

Problem: Add: $4.8 + 11.69$.

Solution:

16.49

Note:
Exercise:

Problem: Add: $5.123 + 18.47$.

Solution:

23.593

Example:
Exercise:

Problem: Subtract: $20 - 14.65$.

Solution:
Solution

	$20-14.65$
Write the numbers so the decimal points line up vertically.	$\begin{array}{r} 20. \\ -14.65 \\ \hline \end{array}$
Remember, 20 is a whole number, so place the decimal point after the 0.	
Put in zeros to the right as placeholders.	$\begin{array}{r} 20.00 \\ -14.65 \\ \hline \end{array}$
Subtract and place the decimal point in the answer.	$\begin{array}{r} \overset{1}{\cancel{2}}\,\overset{\overset{9}{\cancel{10}}}{\cancel{0}}\,.\,\overset{\overset{9}{\cancel{10}}}{\cancel{0}}\,\overset{\cancel{10}}{\cancel{0}} \\ -1\,4\,.\,6\,5 \\ \hline 5\,.\,3\,5 \end{array}$

Note:
Exercise:

Problem: Subtract: $10-9.58$.

Solution:

0.42

Note:
Exercise:

Problem: Subtract: $50-37.42$.

Solution:

12.58

Multiply and Divide Decimals

Multiplying decimals is very much like multiplying whole numbers—we just have to determine where to place the decimal point. The procedure for multiplying decimals will make sense if we first convert them to fractions and then multiply.

So let’s see what we would get as the product of decimals by converting them to fractions first. We will do two examples side-by-side. Look for a pattern!

	$\underbrace{(0.3)}_{1\text{ place}}$ $\underbrace{(0.7)}_{1\text{ place}}$ $\underbrace{(0.2)}_{1\text{ place}}$ $\underbrace{(0.46)}_{2\text{ places}}$
Convert to fractions.	$\frac{3}{10} \cdot \frac{7}{10}$ $\frac{2}{10} \cdot \frac{46}{100}$
Multiply.	$\frac{21}{100}$ $\frac{92}{1000}$
Convert to decimals.	$\underbrace{0.21}_{2\text{ places}}$ $\underbrace{0.092}_{3\text{ places}}$

Notice, in the first example, we multiplied two numbers that each had one digit after the decimal point and the product had two decimal places. In the second example, we multiplied a number with one decimal place by a number with two decimal places and the product had three decimal places.

We multiply the numbers just as we do whole numbers, temporarily ignoring the decimal point. We then count the number of decimal points in the factors and that sum tells us the number of decimal places in the product.

The rules for multiplying positive and negative numbers apply to decimals, too, of course!

When *multiplying* two numbers,

- if their signs are the *same* the product is *positive*.
- if their signs are *different* the product is *negative*.

When we multiply signed decimals, first we determine the sign of the product and then multiply as if the numbers were both positive. Finally, we write the product with the appropriate sign.

Note:
Multiply Decimals.

Determine the sign of the product.
Write in vertical format, lining up the numbers on the right. Multiply the numbers as if they were whole numbers, temporarily ignoring the decimal points.
Place the decimal point. The number of decimal places in the product is the sum of the number of decimal places in the factors.
Write the product with the appropriate sign.

Example:
Exercise:

Problem: Multiply: $(-3.9)\,(4.075)$.

Solution:
Solution

	(−3.9)(4.075)
The signs are different. The product will be negative.	
Write in vertical format, lining up the numbers on the right.	$\begin{array}{r} 4.075 \\ \times\ 3.9 \\ \hline \end{array}$
Multiply.	$\begin{array}{r} 4.075 \\ \times\ 3.9 \\ \hline 36675 \\ 12225 \\ \hline 158925 \end{array}$
Add the number of decimal places in the factors (1 + 3). $\underbrace{(-3.9)}_{\text{1 place}}\ \underbrace{(4.075)}_{\text{3 places}}$ Place the decimal point 4 places from the right.	$\begin{array}{r} 4.075 \\ \times\ 3.9 \\ \hline 36675 \\ 12225 \\ \hline 15.\underbrace{8925}_{\text{4 places}} \end{array}$
The signs are different, so the product is negative.	(−3.9)(4.075) = −15.8925

Note:
Exercise:

Problem: Multiply: $-4.5\,(6.107)$.

Solution:

-27.4815

Note:
Exercise:

Problem: Multiply: $-10.79\,(8.12)$.

Solution:

-87.6148

In many of your other classes, especially in the sciences, you will multiply decimals by powers of 10 (10, 100, 1000, etc.). If you multiply a few products on paper, you may notice a pattern relating the number of zeros in the power of 10 to number of decimal places we move the decimal point to the right to get the product.

Note:
Multiply a Decimal by a Power of Ten.

Move the decimal point to the right the same number of places as the number of zeros in the power of 10. Add zeros at the end of the number as needed.

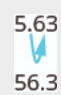
Example:
Exercise:

Problem: Multiply 5.63 ⓐ by 10 ⓑ by 100 ⓒ by 1,000.

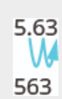
Solution:

By looking at the number of zeros in the multiple of ten, we see the number of places we need to move the decimal to the right.

ⓐ

	5.63(10)
There is 1 zero in 10, so move the decimal point 1 place to the right.	

ⓑ

	5.63(100)
There are 2 zeros in 100, so move the decimal point 2 places to the right.	

ⓒ

	5.63(1,000)
There are 3 zeros in 1,000, so move the decimal point 3 places to the right.	
A zero must be added at the end.	5,630

Note:
Exercise:

Problem: Multiply 2.58 ⓐ by 10 ⓑ by 100 ⓒ by 1,000.

Solution:

ⓐ 25.8 ⓑ 258 ⓒ 2,580

Note:
Exercise:

Problem: Multiply 14.2 ⓐ by 10 ⓑ by 100 ⓒ by 1,000.

Solution:

ⓐ 142 ⓑ 1,420 ⓒ 14,200

Just as with multiplication, division of decimals is very much like dividing whole numbers. We just have to figure out where the decimal point must be placed.

To divide decimals, determine what power of 10 to multiply the denominator by to make it a whole number. Then multiply the numerator by that same power of 10. Because of the equivalent fractions property, we haven't changed the value of the fraction! The effect is to move the decimal points in the numerator and denominator the same number of places to the right. For example:
Equation:

$$\begin{array}{c}\frac{0.8}{0.4}\\ \frac{0.8(10)}{0.4(10)}\\ \frac{8}{4}\end{array}$$

We use the rules for dividing positive and negative numbers with decimals, too. When dividing signed decimals, first determine the sign of the quotient and then divide as if the numbers were both positive. Finally, write the quotient with the appropriate sign.

We review the notation and vocabulary for division:
Equation:

$$\underset{\text{dividend}}{a} \div \underset{\text{divisor}}{b} = \underset{\text{quotient}}{c} \qquad\qquad \underset{\text{divisor}}{b}\overset{\overset{\text{c}}{\text{quotient}}}{\overline{\big)\underset{\text{dividend}}{a}}}$$

We'll write the steps to take when dividing decimals, for easy reference.

Note:
Divide Decimals.

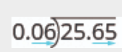
Determine the sign of the quotient.
Make the divisor a whole number by "moving" the decimal point all the way to the right. "Move" the decimal point in the dividend the same number of places—adding zeros as needed.
Divide. Place the decimal point in the quotient above the decimal point in the dividend.
Write the quotient with the appropriate sign.

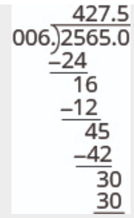
Example:
Exercise:

Problem: Divide: $-25.56 \div (-0.06)$.

Solution:
Solution

Remember, you can "move" the decimals in the divisor and dividend because of the Equivalent Fractions Property.

	$-25.65 \div (-0.06)$
The signs are the same.	The quotient is positive.
Make the divisor a whole number by "moving" the decimal point all the way to the right.	
"Move" the decimal point in the dividend the same number of places.	
Divide. Place the decimal point in the quotient above the decimal point in the dividend.	

	
Write the quotient with the appropriate sign.	$-25.65 \div (-0.06) = 427.5$

Note:
Exercise:

Problem: Divide: $-23.492 \div (-0.04)$.

Solution:

687.3

Note:
Exercise:

Problem: Divide: $-4.11 \div (-0.12)$.

Solution:

34.25

A common application of dividing whole numbers into decimals is when we want to find the price of one item that is sold as part of a multi-pack. For example, suppose a case of 24 water bottles costs \$3.99. To find the price of one water bottle, we would divide \$3.99 by 24. We show this division in [link]. In calculations with money, we will round the answer to the nearest cent (hundredth).

Example:
Exercise:

Problem: Divide: $\$3.99 \div 24$.

Solution:
Solution

	$\$3.99 \div 24$
Place the decimal point in the quotient above the decimal point in the dividend.	
Divide as usual. When do we stop? Since this division involves money, we round it to the nearest cent (hundredth.) To do this, we must carry the division to the thousandths place.	0.166 24)3.990 24 159 144 150 144 6
Round to the nearest cent.	$\$0.166 \approx \$0.17$ $\$3.99 \div 24 \approx \$0.17$

Note:
Exercise:

Problem: Divide: $\$6.99 \div 36$.

Solution:

$0.19

Note:
Exercise:

Problem: Divide: $\$4.99 \div 12$.

Solution:

$0.42

Convert Decimals, Fractions, and Percents

We convert decimals into fractions by identifying the place value of the last (farthest right) digit. In the decimal 0.03 the 3 is in the hundredths place, so 100 is the denominator of the fraction equivalent to 0.03.
Equation:

$$0\ 0.03 = \frac{3}{100}$$

Notice, when the number to the left of the decimal is zero, we get a fraction whose numerator is less than its denominator. Fractions like this are called proper fractions.

The steps to take to convert a decimal to a fraction are summarized in the procedure box.

Note:
Convert a Decimal to a Proper Fraction.

Determine the place value of the final digit.
Write the fraction.

- numerator—the “numbers” to the right of the decimal point
- denominator—the place value corresponding to the final digit

Example:
Exercise:

Problem: Write 0.374 as a fraction.

Solution:
Solution

	0.374
Determine the place value of the final digit.	0.3 7 4 tenths hundredths thousandths
Write the fraction for 0.374: • The numerator is 374. • The denominator is 1,000.	$\frac{374}{1000}$
Simplify the fraction.	$\frac{2\cdot187}{2\cdot500}$
Divide out the common factors.	$\frac{187}{500}$ so, $0.374=\frac{187}{500}$

Did you notice that the number of zeros in the denominator of $\frac{374}{1,000}$ is the same as the number of decimal places in 0.374?

Note:
Exercise:

Problem: Write 0.234 as a fraction.

Solution:

$\frac{117}{500}$

Note:
Exercise:

Problem: Write 0.024 as a fraction.

Solution:

$\frac{3}{125}$

We've learned to convert decimals to fractions. Now we will do the reverse—convert fractions to decimals. Remember that the fraction bar means division. So $\frac{4}{5}$ can be written $4 \div 5$ or $5\overline{)4}$. This leads to the following method for converting a fraction to a decimal.

Note:
Convert a Fraction to a Decimal.
To convert a fraction to a decimal, divide the numerator of the fraction by the denominator of the fraction.

Example:
Exercise:

Problem: Write $-\frac{5}{8}$ as a decimal.

Solution:
Solution

Since a fraction bar means division, we begin by writing $\frac{5}{8}$ as $8\overline{)5}$. Now divide.

$$
\begin{array}{r}
0.625 \\
8\overline{)5.000} \\
\underline{48} \\
20 \\
\underline{16} \\
40 \\
\underline{40}
\end{array}
$$

so, $-\frac{5}{8} = -0.625$

Note:
Exercise:

Problem: Write $-\frac{7}{8}$ as a decimal.

Solution:

-0.875

Note:
Exercise:

Problem: Write $-\frac{3}{8}$ as a decimal.

Solution:

-0.375

When we divide, we will not always get a zero remainder. Sometimes the quotient ends up with a decimal that repeats. A **repeating decimal** is a decimal in which the last digit or group of digits repeats endlessly. A bar is placed over the repeating block of digits to indicate it repeats.

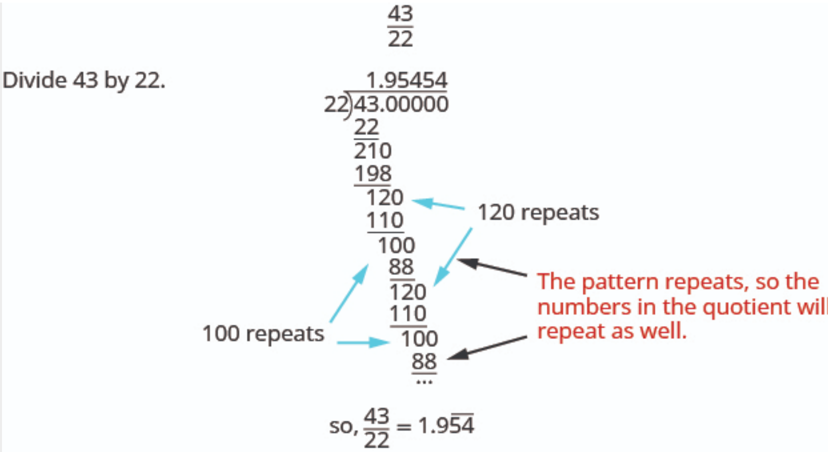
Note:
Repeating Decimal
A **repeating decimal** is a decimal in which the last digit or group of digits repeats endlessly.

A bar is placed over the repeating block of digits to indicate it repeats.

Example:
Exercise:

Problem: Write $\frac{43}{22}$ as a decimal.

Solution:
Solution



Note:
Exercise:

Problem: Write $\frac{27}{11}$ as a decimal.

Solution:

$2.\overline{45}$

Note:
Exercise:

Problem: Write $\frac{51}{22}$ as a decimal.

Solution:

$2.3\overline{18}$

Sometimes we may have to simplify expressions with fractions and decimals together.

Example:
Exercise:

Problem: Simplify: $\frac{7}{8} + 6.4$.

Solution:
Solution

First we must change one number so both numbers are in the same form. We can change the fraction to a decimal, or change the decimal to a fraction. Usually it is easier to change the fraction to a decimal.

		$\frac{7}{8}+6.4$
Change $\frac{7}{8}$ to a decimal.	0.875 8)7.000 64 60 56 40 40 0	
Add.		$0.875+6.4$
		7.275
		So, $\frac{7}{8}+6.4=7.275$

Note:
Exercise:

Problem: Simplify: $\frac{3}{8}+4.9$.

Solution:

5.275

Note:
Exercise:

Problem: Simplify: $5.7+\frac{13}{20}$.

Solution:

6.35

A **percent** is a ratio whose denominator is 100. Percent means per hundred. We use the percent symbol, %, to show percent.

Note:
Percent
A **percent** is a ratio whose denominator is 100.

Since a percent is a ratio, it can easily be expressed as a fraction. Percent means per 100, so the denominator of the fraction is 100. We then change the fraction to a decimal by dividing the numerator by the denominator.
Equation:

	6%	78%	135%
Write as a ratio with denominator 100.	$\frac{6}{100}$	$\frac{78}{100}$	$\frac{135}{100}$
Change the fraction to a decimal by dividing the numerator by the denominator.	0.06	0.78	1.35

Do you see the pattern? *To convert a percent number to a decimal number, we move the decimal point two places to the left.*

Example:
Exercise:

Problem: Convert each percent to a decimal: ⓐ 62% ⓑ 135% ⓒ 35.7%.

Solution:
Solution

ⓐ	
	
Move the decimal point two places to the left.	0.62

ⓑ	

	135%
Move the decimal point two places to the left.	1.35

ⓒ	
	5.7%
Move the decimal point two places to the left.	0.057

Note:
Exercise:

Problem: Convert each percent to a decimal: ⓐ 9% ⓑ 87% ⓒ 3.9%.

Solution:

ⓐ 0.09 ⓑ 0.87 ⓒ 0.039

Note:
Exercise:

Problem: Convert each percent to a decimal: ⓐ 3% ⓑ 91% ⓒ 8.3%.

Solution:

ⓐ 0.03 ⓑ 0.91 ⓒ 0.083

Converting a decimal to a percent makes sense if we remember the definition of percent and keep place value in mind.

To convert a decimal to a percent, remember that percent means per hundred. If we change the decimal to a fraction whose denominator is 100, it is easy to change that fraction to a percent.
Equation:

	0.83	1.05	0.075
Write as a fraction.	$\frac{83}{100}$	$1\frac{5}{100}$	$\frac{75}{1000}$
The denominator is 100.		$\frac{105}{100}$	$\frac{7.5}{100}$
Write the ratio as a percent.	83%	105%	7.5%

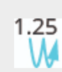
Recognize the pattern? *To convert a decimal to a percent, we move the decimal point two places to the right and then add the percent sign.*

Example:
Exercise:

Problem: Convert each decimal to a percent: ⓐ 0.51 ⓑ 1.25 ⓒ 0.093.

Solution:
Solution

ⓐ	
	
Move the decimal point two places to the right.	51%

ⓑ	
	
Move the decimal point two places to the right.	125%

ⓒ	
	
Move the decimal point two places to the right.	9.3%

Note:
Exercise:

Problem: Convert each decimal to a percent: ⓐ 0.17 ⓑ 1.75 ⓒ 0.0825.

Solution:

ⓐ 17% ⓑ 175% ⓒ 8.25%

Note:
Exercise:

Problem: Convert each decimal to a percent: ⓐ 0.41 ⓑ 2.25 ⓒ 0.0925.

Solution:

ⓐ 41% ⓑ 225% ⓒ 9.25%

Key Concepts

- **Name a Decimal**

  Name the number to the left of the decimal point.
  Write ”and” for the decimal point.
  Name the “number” part to the right of the decimal point as if it were a whole number.
  Name the decimal place of the last digit.

- **Write a Decimal**

  Look for the word ‘and’—it locates the decimal point. Place a decimal point under the word ‘and.’ Translate the words before ‘and’ into the whole number and place it to the left of the decimal point. If there is no “and,” write a “0” with a decimal point to its right.
  Mark the number of decimal places needed to the right of the decimal point by noting the place value indicated by the last word.
  Translate the words after ‘and’ into the number to the right of the decimal point. Write the number in the spaces—putting the final digit in the last place.
  Fill in zeros for place holders as needed.

- **Round a Decimal**

Locate the given place value and mark it with an arrow.
Underline the digit to the right of the place value.
Is this digit greater than or equal to 5? Yes—add 1 to the digit in the given place value. No—do not change the digit in the given place value.
Rewrite the number, deleting all digits to the right of the rounding digit.

- **Add or Subtract Decimals**

  Write the numbers so the decimal points line up vertically.
  Use zeros as place holders, as needed.
  Add or subtract the numbers as if they were whole numbers. Then place the decimal in the answer under the decimal points in the given numbers.

- **Multiply Decimals**

  Determine the sign of the product.
  Write in vertical format, lining up the numbers on the right. Multiply the numbers as if they were whole numbers, temporarily ignoring the decimal points.
  Place the decimal point. The number of decimal places in the product is the sum of the decimal places in the factors.
  Write the product with the appropriate sign.

- **Multiply a Decimal by a Power of Ten**

  Move the decimal point to the right the same number of places as the number of zeros in the power of 10.
  Add zeros at the end of the number as needed.

- **Divide Decimals**

  Determine the sign of the quotient.
  Make the divisor a whole number by “moving” the decimal point all the way to the right. “Move” the decimal point in the dividend the same number of places - adding zeros as needed.
  Divide. Place the decimal point in the quotient above the decimal point in the dividend.
  Write the quotient with the appropriate sign.

- **Convert a Decimal to a Proper Fraction**

  Determine the place value of the final digit.
  Write the fraction: numerator—the ‘numbers’ to the right of the decimal point; denominator—the place value corresponding to the final digit.

- **Convert a Fraction to a Decimal** Divide the numerator of the fraction by the denominator.

Practice Makes Perfect

Name and Write Decimals

In the following exercises, write as a decimal.
Exercise:

Problem: Twenty-nine and eighty-one hundredths

Solution:

29.81

Exercise:

Problem: Sixty-one and seventy-four hundredths

Exercise:

Problem: Seven tenths

Solution:

0.7

Exercise:

Problem: Six tenths

Exercise:

Problem: Twenty-nine thousandth

Solution:

0.029

Exercise:

Problem: Thirty-five thousandths

Exercise:

Problem: Negative eleven and nine ten-thousandths

Solution:

-11.0009

Exercise:

Problem: Negative fifty-nine and two ten-thousandths

In the following exercises, name each decimal.

Exercise:

Problem: 5.5

Solution:

five and five tenths

Exercise:

Problem: 14.02

Exercise:

Problem: 8.71

Solution:

eight and seventy-one hundredths

Exercise:

Problem: 2.64

Exercise:

Problem: 0.002

Solution:

two thousandths

Exercise:

Problem: 0.479

Exercise:

Problem: -17.9

Solution:

negative seventeen and nine tenths

Exercise:

Problem: -31.4

Round Decimals

In the following exercises, round each number to the nearest tenth.

Exercise:

Problem: 0.67

Solution:

0.7

Exercise:

Problem: 0.49

Exercise:

Problem: 2.84

Solution:

2.8

Exercise:

Problem: 4.63

In the following exercises, round each number to the nearest hundredth.

Exercise:

Problem: 0.845

Solution:

0.85

Exercise:

Problem: 0.761

Exercise:

Problem: 0.299

Solution:

0.30

Exercise:

Problem: 0.697

Exercise:

Problem: 4.098

Solution:

4.10

Exercise:

Problem: 7.096

In the following exercises, round each number to the nearest ⓐ hundredth ⓑ tenth ⓒ whole number.

Exercise:

Problem: 5.781

Solution:

ⓐ 5.78 ⓑ 5.8 ⓒ 6

Exercise:

Problem: 1.6381

Exercise:

Problem: 63.479

Solution:

ⓐ 63.48 ⓑ 63.5 ⓒ 63

Exercise:

Problem: 84.281

Add and Subtract Decimals

In the following exercises, add or subtract.
Exercise:

Problem: $16.92 + 7.56$

Solution:

24.48

Exercise:

Problem: $248.25 - 91.29$

Exercise:

Problem: $21.76 - 30.99$

Solution:

-9.23

Exercise:

Problem: $38.6 + 13.67$

Exercise:

Problem: $-16.53 - 24.38$

Solution:

-40.91

Exercise:

Problem: $-19.47 - 32.58$

Exercise:

Problem: $-38.69 + 31.47$

Solution:

-7.22

Exercise:

Problem: $29.83 + 19.76$

Exercise:

Problem: $72.5 - 100$

Solution:

-27.5

Exercise:

Problem: $86.2 - 100$

Exercise:

Problem: $15 + 0.73$

Solution:

15.73

Exercise:

Problem: $27 + 0.87$

Exercise:

Problem: $91.95 - (-10.462)$

Solution:

102.212

Exercise:

Problem: $94.69 - (-12.678)$

Exercise:

Problem: $55.01 - 3.7$

Solution:

51.31

Exercise:

Problem: $59.08 - 4.6$

Exercise:

Problem: $2.51 - 7.4$

Solution:

-4.89

Exercise:

Problem: $3.84 - 6.1$

Multiply and Divide Decimals

In the following exercises, multiply.

Exercise:

Problem: $(0.24)(0.6)$

Solution:

0.144

Exercise:

Problem: $(0.81)(0.3)$

Exercise:

Problem: $(5.9)(7.12)$

Solution:

42.008

Exercise:

Problem: $(2.3)(9.41)$

Exercise:

Problem: $(-4.3)(2.71)$

Solution:

-11.653

Exercise:

Problem: $(-8.5)(1.69)$

Exercise:

Problem: $(-5.18)(-65.23)$

Solution:

337.8914

Exercise:

Problem: $(-9.16)(-68.34)$

Exercise:

Problem: $(0.06)(21.75)$

Solution:

1.305

Exercise:

Problem: $(0.08)\,(52.45)$

Exercise:

Problem: $(9.24)\,(10)$

Solution:

92.4

Exercise:

Problem: $(6.531)\,(10)$

Exercise:

Problem: $(55.2)\,(1000)$

Solution:

55,200

Exercise:

Problem: $(99.4)\,(1000)$

In the following exercises, divide.

Exercise:

Problem: $4.75 \div 25$

Solution:

0.19

Exercise:

Problem: $12.04 \div 43$

Exercise:

Problem: $\$117.25 \div 48$

Solution:

\$2.44

Exercise:

Problem: $\$109.24 \div 36$

Exercise:

Problem: $0.6 \div 0.2$

Solution:

3

Exercise:

Problem: $0.8 \div 0.4$

Exercise:

Problem: $1.44 \div (-0.3)$

Solution:

-4.8

Exercise:

Problem: $1.25 \div (-0.5)$

Exercise:

Problem: $-1.75 \div (-0.05)$

Solution:

35

Exercise:

Problem: $-1.15 \div (-0.05)$

Exercise:

Problem: $5.2 \div 2.5$

Solution:

2.08

Exercise:

Problem: $6.5 \div 3.25$

Exercise:

Problem: $11 \div 0.55$

Solution:

20

Exercise:

Problem: $14 \div 0.35$

Convert Decimals, Fractions and Percents

In the following exercises, write each decimal as a fraction.

Exercise:

Problem: 0.04

Solution:

$\frac{1}{25}$

Exercise:

Problem: 0.19

Exercise:

Problem: 0.52

Solution:

$\frac{13}{25}$

Exercise:

Problem: 0.78

Exercise:

Problem: 1.25

Solution:

$\frac{5}{4}$

Exercise:

Problem: 1.35

Exercise:

Problem: 0.375

Solution:

$\frac{3}{8}$

Exercise:

Problem: 0.464

Exercise:

Problem: 0.095

Solution:

$\frac{19}{200}$

Exercise:

Problem: 0.085

In the following exercises, convert each fraction to a decimal.

Exercise:

Problem: $\frac{17}{20}$

Solution:

0.85

Exercise:

Problem: $\frac{13}{20}$

Exercise:

Problem: $\frac{11}{4}$

Solution:

2.75

Exercise:

Problem: $\frac{17}{4}$

Exercise:

Problem: $-\frac{310}{25}$

Solution:

-12.4

Exercise:

Problem: $-\frac{284}{25}$

Exercise:

Problem: $\frac{15}{11}$

Solution:

$1.\overline{36}$

Exercise:

Problem: $\frac{18}{11}$

Exercise:

Problem: $\frac{15}{111}$

Solution:

$0.1\overline{35}$

Exercise:

Problem: $\frac{25}{111}$

Exercise:

Problem: $2.4 + \frac{5}{8}$

Solution:

3.025

Exercise:

Problem: $3.9 + \frac{9}{20}$

In the following exercises, convert each percent to a decimal.

Exercise:

Problem: 1%

Solution:

0.011

Exercise:

Problem: 2%

Exercise:

Problem: 63%

Solution:

0.63

Exercise:

Problem: 71%

Exercise:

Problem: 150%

Solution:

1.5

Exercise:

Problem: 250%

Exercise:

Problem: 21.4%

Solution:

0.214

Exercise:

Problem: 39.3%

Exercise:

Problem: 7.8%

Solution:

0.078

Exercise:

Problem: 6.4%

In the following exercises, convert each decimal to a percent.

Exercise:

Problem: 0.01

Solution:

1%

Exercise:

Problem: 0.03

Exercise:

Problem: 1.35

Solution:

135%

Exercise:

Problem: 1.56

Exercise:

Problem: 3

Solution:

300%

Exercise:

Problem: 4

Exercise:

Problem: 0.0875

Solution:

8.75%

Exercise:

Problem: 0.0625

Exercise:

Problem: 2.254

Solution:

225.4%

Exercise:

Problem: 2.317

Everyday Math

Exercise:

Problem:

Salary Increase Danny got a raise and now makes $58,965.95 a year. Round this number to the nearest
ⓐ dollar
ⓑ thousand dollars
ⓒ ten thousand dollars.

Solution:

ⓐ $58,966 ⓑ $59,000 ⓒ $60,000

Exercise:

Problem: **New Car Purchase** Selena's new car cost $23,795.95. Round this number to the nearest
ⓐ dollar
ⓑ thousand dollars
ⓒ ten thousand dollars.

Exercise:

Problem:

Sales Tax Hyo Jin lives in San Diego. She bought a refrigerator for $1,624.99 and when the clerk calculated the sales tax it came out to exactly $142.186625. Round the sales tax to the nearest
ⓐ penny and
ⓑ dollar.

Solution:

ⓐ $142.19; ⓑ $142

Exercise:

Problem:

Sales Tax Jennifer bought a $1,038.99 dining room set for her home in Cincinnati. She calculated the sales tax to be exactly $67.53435. Round the sales tax to the nearest
ⓐ penny and
ⓑ dollar.

Exercise:

Problem:

Paycheck Annie has two jobs. She gets paid $14.04 per hour for tutoring at City College and $8.75 per hour at a coffee shop. Last week she tutored for 8 hours and worked at the coffee shop for 15 hours.
ⓐ How much did she earn?
ⓑ If she had worked all 23 hours as a tutor instead of working both jobs, how much more would she have earned?

Solution:

ⓐ $243.57 ⓑ $79.35

Exercise:

Problem:

Paycheck Jake has two jobs. He gets paid $7.95 per hour at the college cafeteria and $20.25 at the art gallery. Last week he worked 12 hours at the cafeteria and 5 hours at the art gallery.
ⓐ How much did he earn?
ⓑ If he had worked all 17 hours at the art gallery instead of working both jobs, how much more would he have earned?

Writing Exercises

Exercise:

Problem: How does knowing about US money help you learn about decimals?

Solution:

Answers may vary

Exercise:

Problem: Explain how you write “three and nine hundredths” as a decimal.

Exercise:

Problem:

Without solving the problem “44 is 80% of what number” think about what the solution might be. Should it be a number that is greater than 44 or less than 44? Explain your reasoning.

Solution:

Answers may vary

Exercise:

Problem:

When the Szetos sold their home, the selling price was 500% of what they had paid for the house 30 years ago. Explain what 500% means in this context.

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
name and write decimals.			
round decimals.			
add and subtract decimals.			
multiply and divide decimals.			
convert decimals, fractions, and percents.			

ⓑ What does this checklist tell you about your mastery of this section? What steps will you take to improve?

Glossary

decimal
: A decimal is another way of writing a fraction whose denominator is a power of ten.

percent
: A percent is a ratio whose denominator is 100.

repeating decimal
: A repeating decimal is a decimal in which the last digit or group of digits repeats endlessly.

The Real Numbers
By the end of this section, you will be able to:

- Simplify expressions with square roots
- Identify integers, rational numbers, irrational numbers, and real numbers
- Locate fractions on the number line
- Locate decimals on the number line

Note:
A more thorough introduction to the topics covered in this section can be found in the *Prealgebra* chapters, **Decimals** and **Properties of Real Numbers**.

Simplify Expressions with Square Roots

Remember that when a number *n* is multiplied by itself, we write n^2 and read it "n squared." The result is called the **square** of *n*. For example,
Equation:

$$\begin{array}{ll} 8^2 & \text{read '8 squared'} \\ 64 & \text{64 is called the } \textit{square} \text{ of 8.} \end{array}$$

Similarly, 121 is the square of 11, because 11^2 is 121.

Note:
Square of a Number
If $n^2 = m$, then *m* is the **square** of *n*.

Note: Doing the Manipulative Mathematics activity "Square Numbers" will help you develop a better understanding of perfect square numbers.

Complete the following table to show the squares of the counting numbers 1 through 15.

Number	n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Square	n^2								64			121				

The numbers in the second row are called perfect square numbers. It will be helpful to learn to recognize the perfect square numbers.

The squares of the counting numbers are positive numbers. What about the squares of negative numbers? We know that when the signs of two numbers are the same, their product is positive. So the square of any negative number is also positive.
Equation:

$$(-3)^2 = 9 \qquad (-8)^2 = 64 \qquad (-11)^2 = 121 \qquad (-15)^2 = 225$$

Did you notice that these squares are the same as the squares of the positive numbers?

Sometimes we will need to look at the relationship between numbers and their squares in reverse. Because $10^2 = 100$, we say 100 is the square of 10. We also say that 10 is a *square root* of 100. A number whose square is m is called a **square root** of *m*.

Note:
Square Root of a Number
If $n^2 = m$, then *n* is a **square root** of *m*.

Notice $(-10)^2 = 100$ also, so -10 is also a square root of 100. Therefore, both 10 and -10 are square roots of 100.

So, every positive number has two square roots—one positive and one negative. What if we only wanted the positive square root of a positive number? The radical sign, $\sqrt{m}$, denotes the positive square root. The positive square root is called the principal square root. When we use the radical sign that always means we want the principal square root.

We also use the radical sign for the square root of zero. Because $0^2 = 0$, $\sqrt{0} = 0$. Notice that zero has only one square root.

Note:
Square Root Notation
$\sqrt{m}$ is read "the square root of *m*"

If $m = n^2$, then $\sqrt{m} = n$, for $n \geq 0$.
The square root of *m*, $\sqrt{m}$, is the positive number whose square is *m*.

Since 10 is the principal square root of 100, we write $\sqrt{100} = 10$. You may want to complete the following table to help you recognize square roots.

$\sqrt{1}$	$\sqrt{4}$	$\sqrt{9}$	$\sqrt{16}$	$\sqrt{25}$	$\sqrt{36}$	$\sqrt{49}$	$\sqrt{64}$	$\sqrt{81}$	$\sqrt{100}$	$\sqrt{121}$	$\sqrt{144}$	$\sqrt{169}$	$\sqrt{196}$	$\sqrt{225}$
									10					

Example:
Exercise:

Problem: Simplify: ⓐ $\sqrt{25}$ ⓑ $\sqrt{121}$.

Solution:
Solution

ⓐ

	$\sqrt{25}$
Since $5^2 = 25$	5

ⓑ

	$\sqrt{121}$
Since $11^2 = 121$	11

Note:
Exercise:

Problem: Simplify: ⓐ $\sqrt{36}$ ⓑ $\sqrt{169}$.

Solution:

ⓐ 6 ⓑ 13

Note:
Exercise:

Problem: Simplify: ⓐ $\sqrt{16}$ ⓑ $\sqrt{196}$.

Solution:

ⓐ 4 ⓑ 14

We know that every positive number has two square roots and the radical sign indicates the positive one. We write $\sqrt{100} = 10$. If we want to find the negative square root of a number, we place a negative in front of the radical sign. For example, $-\sqrt{100} = -10$. We read $-\sqrt{100}$ as "the opposite of the square root of 10."

Example:
Exercise:

Problem: Simplify: ⓐ $-\sqrt{9}$ ⓑ $-\sqrt{144}$.

Solution:

ⓐ

	$-\sqrt{9}$
The negative is in front of the radical sign.	-3

ⓑ

	$-\sqrt{144}$
The negative is in front of the radical sign.	-12

Note:
Exercise:

Problem: Simplify: ⓐ $-\sqrt{4}$ ⓑ $-\sqrt{225}$.

Solution:

ⓐ -2 ⓑ -15

Note:
Exercise:

Problem: Simplify: ⓐ $-\sqrt{81}$ ⓑ $-\sqrt{100}$.

Solution:

ⓐ -9 ⓑ -10

Identify Integers, Rational Numbers, Irrational Numbers, and Real Numbers

We have already described numbers as *counting numbers*, *whole numbers*, and *integers*. What is the difference between these types of numbers?
Equation:

$$\begin{array}{ll}\text{Counting numbers} & 1, 2, 3, 4, \ldots \\ \text{Whole numbers} & 0, 1, 2, 3, 4, \ldots \\ \text{Integers} & \ldots -3, -2, -1, 0, 1, 2, 3, \ldots\end{array}$$

What type of numbers would we get if we started with all the integers and then included all the fractions? The numbers we would have form the set of rational numbers. A **rational number** is a number that can be written as a ratio of two integers.

Note:
Rational Number
A **rational number** is a number of the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.
A rational number can be written as the ratio of two integers.

All signed fractions, such as $\frac{4}{5}, -\frac{7}{8}, \frac{13}{4}, -\frac{20}{3}$ are rational numbers. Each numerator and each denominator is an integer.

Are integers rational numbers? To decide if an integer is a rational number, we try to write it as a ratio of two integers. Each integer can be written as a ratio of integers in many ways. For example, 3 is equivalent to $\frac{3}{1}, \frac{6}{2}, \frac{9}{3}, \frac{12}{4}, \frac{15}{5}\ldots$

An easy way to write an integer as a ratio of integers is to write it as a fraction with denominator one.

Equation:

$$3=\frac{3}{1} \qquad -8=-\frac{8}{1} \qquad 0=\frac{0}{1}$$

Since any integer can be written as the ratio of two integers, *all integers are rational numbers*! Remember that the counting numbers and the whole numbers are also integers, and so they, too, are rational.

What about decimals? Are they rational? Let's look at a few to see if we can write each of them as the ratio of two integers.

We've already seen that integers are rational numbers. The integer -8 could be written as the decimal -8.0. So, clearly, some decimals are rational.

Think about the decimal 7.3. Can we write it as a ratio of two integers? Because 7.3 means $7\frac{3}{10}$, we can write it as an improper fraction, $\frac{73}{10}$. So 7.3 is the ratio of the integers 73 and 10. It is a rational number.

In general, any decimal that ends after a number of digits (such as 7.3 or -1.2684) is a rational number. We can use the place value of the last digit as the denominator when writing the decimal as a fraction.

Example:

Exercise:

Problem: Write as the ratio of two integers: ⓐ -27 ⓑ 7.31.

Solution:

ⓐ

	-27
Write it as a fraction with denominator 1.	$\frac{-27}{1}$

ⓑ

	7.31
Write is as a mixed number. Remember. 7 is the whole number and the decimal part, 0.31, indicates hundredths.	$7\frac{31}{100}$
Convert to an improper fraction.	$\frac{731}{100}$

So we see that -27 and 7.31 are both rational numbers, since they can be written as the ratio of two integers.

Note:
Exercise:

Problem: Write as the ratio of two integers: ⓐ -24 ⓑ 3.57.

Solution:

ⓐ $\frac{-24}{1}$ ⓑ $\frac{357}{100}$

Note:
Exercise:

Problem: Write as the ratio of two integers: ⓐ -19 ⓑ 8.41.

Solution:

ⓐ $\frac{-19}{1}$ ⓑ $\frac{841}{100}$

Let's look at the decimal form of the numbers we know are rational.

We have seen that *every integer is a rational number*, since $a = \frac{a}{1}$ for any integer, a. We can also change any integer to a decimal by adding a decimal point and a zero.
Equation:

$$\begin{array}{lllllll} \text{Integer} & -2 & -1 & 0 & 1 & 2 & 3 \\ \text{Decimal form} & -2.0 & -1.0 & 0.0 & 1.0 & 2.0 & 3.0 \end{array}$$

Equation:

$$\text{These decimal numbers stop.}$$

We have also seen that *every fraction is a rational number*. Look at the decimal form of the fractions we considered above.
Equation:

$$\begin{array}{lllll} \text{Ratio of integers} & \frac{4}{5} & -\frac{7}{8} & \frac{13}{4} & -\frac{20}{3} \\ \text{The decimal form} & 0.8 & -0.875 & 3.25 & \begin{array}{l} -6.666\ldots \\ -6.\overline{6} \end{array} \end{array}$$

Equation:

$$\text{These decimals either stop or repeat.}$$

What do these examples tell us?

Every rational number can be written both as a ratio of integers, $\left(\frac{p}{q}\right.$*, where p and q are integers and* $\left.q \neq 0\right)$*, and as a decimal that either stops or repeats.*

Here are the numbers we looked at above expressed as a ratio of integers and as a decimal:

Fractions					**Integers**					
Number	$\frac{4}{5}$	$-\frac{7}{8}$	$\frac{13}{4}$	$-\frac{20}{3}$	-2	-1	0	1	2	3
Ratio of Integers	$\frac{4}{5}$	$-\frac{7}{8}$	$\frac{13}{4}$	$-\frac{20}{3}$	$-\frac{2}{1}$	$-\frac{1}{1}$	$\frac{0}{1}$	$\frac{1}{1}$	$\frac{2}{1}$	$\frac{3}{1}$
Decimal Form	0.8	-0.875	3.25	$-6.\overline{6}$	-2.0	-1.0	0.0	1.0	2.0	3.0

Note:
Rational Number
A **rational number** is a number of the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.
Its decimal form stops or repeats.

Are there any decimals that do not stop or repeat? Yes!

The number π (the Greek letter *pi*, pronounced “pie”), which is very important in describing circles, has a decimal form that does not stop or repeat.
Equation:

$$\pi = 3.141592654...$$

We can even create a decimal pattern that does not stop or repeat, such as
Equation:

$$2.01001000100001...$$

Numbers whose decimal form does not stop or repeat cannot be written as a fraction of integers. We call these numbers irrational.

Note:
Irrational Number
An **irrational number** is a number that cannot be written as the ratio of two integers.
Its decimal form does not stop and does not repeat.

Let’s summarize a method we can use to determine whether a number is rational or irrational.

Note:
Rational or Irrational?
If the decimal form of a number

- *repeats or stops*, the number is **rational**.
- *does not repeat and does not stop*, the number is **irrational**.

Example:
Exercise:

Problem:

Given the numbers $0.58\overline{3}, 0.47, 3.605551275\ldots$ list the ⓐ rational numbers ⓑ irrational numbers.

Solution:
Solution

ⓐ
Look for decimals that repeat or stop.

The 3 repeats in $0.58\overline{3}$.
The decimal 0.47 stops after the 7.
So $0.58\overline{3}$ and 0.47 are rational.

ⓑ
Look for decimals that neither stop nor repeat.

$3.605551275\ldots$ has no repeating block of digits and it does not stop.
So $3.605551275\ldots$ is irrational.

Note:
Exercise:

Problem:

For the given numbers list the ⓐ rational numbers ⓑ irrational numbers: $0.29, 0.81\overline{6}, 2.515115111\ldots$.

Solution:

ⓐ $0.29, 0.81\overline{6}$ ⓑ $2.515115111\ldots$

Note:
Exercise:

Problem:

For the given numbers list the ⓐ rational numbers ⓑ irrational numbers: $2.6\overline{3}, 0.125, 0.418302\ldots$

Solution:

ⓐ $2.6\overline{3}, 0.125$ ⓑ $0.418302\ldots$

Example:
Exercise:

Problem: For each number given, identify whether it is rational or irrational: ⓐ $\sqrt{36}$ ⓑ $\sqrt{44}$.

Solution:

ⓐ Recognize that 36 is a perfect square, since $6^2 = 36$. So $\sqrt{36} = 6$, therefore $\sqrt{36}$ is rational.
ⓑ Remember that $6^2 = 36$ and $7^2 = 49$, so 44 is not a perfect square. Therefore, the decimal form of $\sqrt{44}$ will never repeat and never stop, so $\sqrt{44}$ is irrational.

Note:
Exercise:

Problem: For each number given, identify whether it is rational or irrational: ⓐ $\sqrt{81}$ ⓑ $\sqrt{17}$.

Solution:

ⓐ rational ⓑ irrational

Note:
Exercise:

Problem: For each number given, identify whether it is rational or irrational: ⓐ $\sqrt{116}$ ⓑ $\sqrt{121}$.

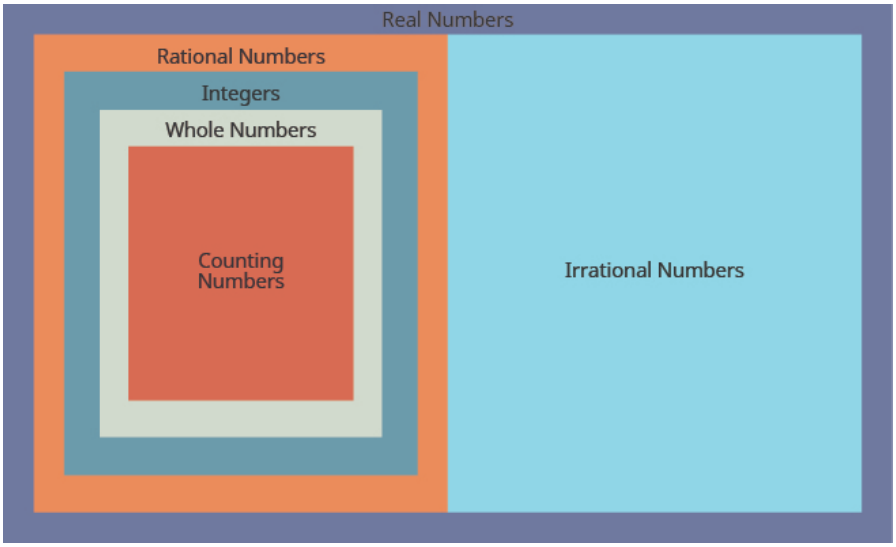
Solution:

ⓐ irrational ⓑ rational

We have seen that all counting numbers are whole numbers, all whole numbers are integers, and all integers are rational numbers. The irrational numbers are numbers whose decimal form does not stop and does not repeat. When we put together the rational numbers and the irrational numbers, we get the set of **real numbers**.

Note:
Real Number
A **real number** is a number that is either rational or irrational.

All the numbers we use in elementary algebra are real numbers. [link] illustrates how the number sets we've discussed in this section fit together.



This chart shows the number sets that make up the set of real numbers. Does the term “real numbers” seem strange to you? Are there any numbers that are not “real,” and, if so, what could they be?

Can we simplify $\sqrt{-25}$? Is there a number whose square is -25?

Equation:

$$(\ \ )^2 = -25?$$

None of the numbers that we have dealt with so far has a square that is -25. Why? Any positive number squared is positive. Any negative number squared is positive. So we say there is no real number equal to $\sqrt{-25}$.

The square root of a negative number is not a real number.

Example:
Exercise:

Problem:

For each number given, identify whether it is a real number or not a real number: ⓐ $\sqrt{-169}$ ⓑ $-\sqrt{64}$.

Solution:

ⓐ There is no real number whose square is -169. Therefore, $\sqrt{-169}$ is not a real number.
ⓑ Since the negative is in front of the radical, $-\sqrt{64}$ is -8, Since -8 is a real number, $-\sqrt{64}$ is a real number.

Note:
Exercise:

Problem:

For each number given, identify whether it is a real number or not a real number: ⓐ $\sqrt{-196}$ ⓑ $-\sqrt{81}$.

Solution:

ⓐ not a real number ⓑ real number

Note:
Exercise:

Problem:

For each number given, identify whether it is a real number or not a real number: ⓐ $-\sqrt{49}$ ⓑ $\sqrt{-121}$.

Solution:

ⓐ real number ⓑ not a real number

Example:
Exercise:

Problem:

Given the numbers $-7, \frac{14}{5}, 8, \sqrt{5}, 5.9, -\sqrt{64}$, list the ⓐ whole numbers ⓑ integers ⓒ rational numbers ⓓ irrational numbers ⓔ real numbers.

Solution:

ⓐ Remember, the whole numbers are 0, 1, 2, 3, … and 8 is the only whole number given.
ⓑ The integers are the whole numbers, their opposites, and 0. So the whole number 8 is an integer, and -7 is the opposite of a whole number so it is an integer, too. Also, notice that 64 is the square of 8 so $-\sqrt{64} = -8$. So the integers are $-7, 8, -\sqrt{64}$.
ⓒ Since all integers are rational, then $-7, 8, -\sqrt{64}$ are rational. Rational numbers also include fractions and decimals that repeat or stop, so $\frac{14}{5}$ and 5.9 are rational. So the list of rational numbers is $-7, \frac{14}{5}, 8, 5.9, -\sqrt{64}$.
ⓓ Remember that 5 is not a perfect square, so $\sqrt{5}$ is irrational.
ⓔ All the numbers listed are real numbers.

Note:
Exercise:

Problem:

For the given numbers, list the ⓐ whole numbers ⓑ integers ⓒ rational numbers ⓓ irrational numbers ⓔ real numbers: $-3, -\sqrt{2}, 0.\bar{3}, \frac{9}{5}, 4, \sqrt{49}$.

Solution:

ⓐ $4, \sqrt{49}$ ⓑ $-3, 4, \sqrt{49}$ ⓒ $-3, 0.\overline{3}, \frac{9}{5}, 4, \sqrt{49}$ ⓓ $-\sqrt{2}$ ⓔ $-3, -\sqrt{2}, 0.\overline{3}, \frac{9}{5}, 4, \sqrt{49}$

Note:
Exercise:

Problem:

For the given numbers, list the ⓐ whole numbers ⓑ integers ⓒ rational numbers ⓓ irrational numbers ⓔ real numbers: $-\sqrt{25}, -\frac{3}{8}, -1, 6, \sqrt{121}, 2.041975\ldots$

Solution:

ⓐ $6, \sqrt{121}$ ⓑ $-\sqrt{25}, -1, 6, \sqrt{121}$ ⓒ $-\sqrt{25}, -\frac{3}{8}, -1, 6, \sqrt{121}$ ⓓ $2.041975\ldots$ ⓔ $-\sqrt{25}, -\frac{3}{8}, -1, 6, \sqrt{121}, 2.041975\ldots$

Locate Fractions on the Number Line

The last time we looked at the number line, it only had positive and negative integers on it. We now want to include fractions and decimals on it.

Note: Doing the Manipulative Mathematics activity “Number Line Part 3” will help you develop a better understanding of the location of fractions on the number line.

Let’s start with fractions and locate $\frac{1}{5}, -\frac{4}{5}, 3, \frac{7}{4}, -\frac{9}{2}, -5,$ and $\frac{8}{3}$ on the number line.

We’ll start with the whole numbers 3 and -5. because they are the easiest to plot. See [link].

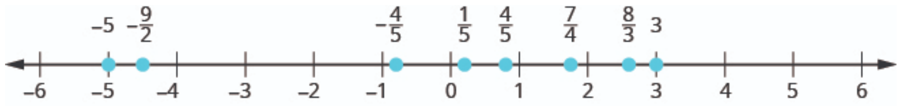
The proper fractions listed are $\frac{1}{5}$ and $-\frac{4}{5}$. We know the proper fraction $\frac{1}{5}$ has value less than one and so would be located between 0 and 1. The denominator is 5, so we divide the unit from 0 to 1 into 5 equal parts $\frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}$. We plot $\frac{1}{5}$. See [link].

Similarly, $-\frac{4}{5}$ is between 0 and -1. After dividing the unit into 5 equal parts we plot $-\frac{4}{5}$. See [link].

Finally, look at the improper fractions $\frac{7}{4}, -\frac{9}{2}, \frac{8}{3}$. These are fractions in which the numerator is greater than the denominator. Locating these points may be easier if you change each of them to a mixed number. See [link].
Equation:

$$\frac{7}{4} = 1\frac{3}{4} \qquad -\frac{9}{2} = -4\frac{1}{2} \qquad \frac{8}{3} = 2\frac{2}{3}$$

[link] shows the number line with all the points plotted.



Example:
Exercise:

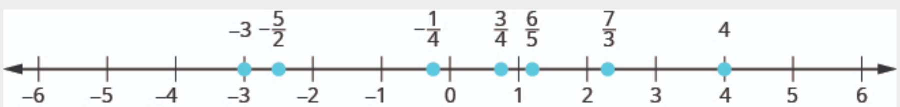
Problem: Locate and label the following on a number line: $4, \frac{3}{4}, -\frac{1}{4}, -3, \frac{6}{5}, -\frac{5}{2},$ and $\frac{7}{3}$.

Solution:

Locate and plot the integers, $4, -3$.

Locate the proper fraction $\frac{3}{4}$ first. The fraction $\frac{3}{4}$ is between 0 and 1. Divide the distance between 0 and 1 into four equal parts then, we plot $\frac{3}{4}$. Similarly plot $-\frac{1}{4}$.

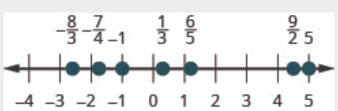
Now locate the improper fractions $\frac{6}{5}, -\frac{5}{2}, \frac{7}{3}$. It is easier to plot them if we convert them to mixed numbers and then plot them as described above: $\frac{6}{5} = 1\frac{1}{5}, -\frac{5}{2} = -2\frac{1}{2}, \frac{7}{3} = 2\frac{1}{3}$.



Note:
Exercise:

Problem: Locate and label the following on a number line: $-1, \frac{1}{3}, \frac{6}{5}, -\frac{7}{4}, \frac{9}{2}, 5, -\frac{8}{3}$.

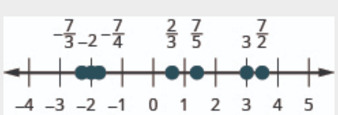
Solution:



Note:
Exercise:

Problem: Locate and label the following on a number line: $-2, \frac{2}{3}, \frac{7}{5}, -\frac{7}{4}, \frac{7}{2}, 3, -\frac{7}{3}$.

Solution:



In [link], we'll use the inequality symbols to order fractions. In previous chapters we used the number line to order numbers.

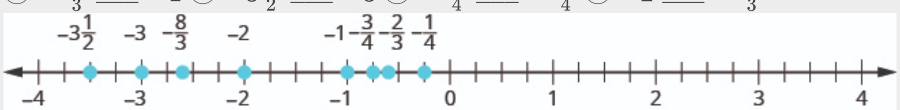
- $a < b$ "a is less than b" when a is to the left of b on the number line
- $a > b$ "a is greater than b" when a is to the right of b on the number line

As we move from left to right on a number line, the values increase.

Example:
Exercise:

Problem: Order each of the following pairs of numbers, using < or >. It may be helpful to refer [link].

ⓐ $-\frac{2}{3}$ ___ -1 ⓑ $-3\frac{1}{2}$ ___ -3 ⓒ $-\frac{3}{4}$ ___ $-\frac{1}{4}$ ⓓ -2 ___ $-\frac{8}{3}$



Solution:

Be careful when ordering negative numbers.

ⓐ

$-\frac{2}{3}$ ___ -1

$-\frac{2}{3}$ is to the right of -1 on the number line. $-\frac{2}{3} > -1$

ⓑ

$-3\frac{1}{2}$ ___ -3

$-3\frac{1}{2}$ is to the left of -3 on the number line. $-3\frac{1}{2} < -3$

ⓒ

$-\frac{3}{4}$ ___ $-\frac{1}{4}$

$-\frac{3}{4}$ is to the left of $-\frac{1}{4}$ on the number line. $-\frac{3}{4} < -\frac{1}{4}$

ⓓ

-2 ___ $-\frac{8}{3}$

-2 is to the right of $-\frac{8}{3}$ on the number line. $-2 > -\frac{8}{3}$

Note:
Exercise:

Problem: Order each of the following pairs of numbers, using < or >:

ⓐ $-\frac{1}{3}$ ___ -1 ⓑ $-1\frac{1}{2}$ ___ -2 ⓒ $-\frac{2}{3}$ ___ $-\frac{1}{3}$ ⓓ -3 ___ $-\frac{7}{3}$.

Solution:

ⓐ $>$ ⓑ $>$ ⓒ $<$ ⓓ $<$

Note:
Exercise:

Problem: Order each of the following pairs of numbers, using $<$ or $>$:

ⓐ -1 ___ $-\frac{2}{3}$ ⓑ $-2\frac{1}{4}$ ___ -2 ⓒ $-\frac{3}{5}$ ___ $-\frac{4}{5}$ ⓓ -4 ___ $-\frac{10}{3}$.

Solution:

ⓐ $<$ ⓑ $<$ ⓒ $>$ ⓓ $<$

Locate Decimals on the Number Line

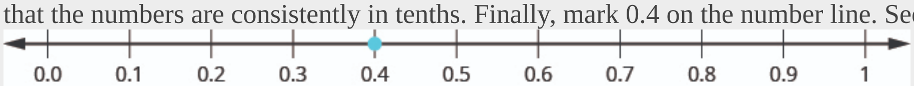
Since decimals are forms of fractions, locating decimals on the number line is similar to locating fractions on the number line.

Example:
Exercise:

Problem: Locate 0.4 on the number line.

Solution:

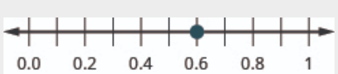
A proper fraction has value less than one. The decimal number 0.4 is equivalent to $\frac{4}{10}$, a proper fraction, so 0.4 is located between 0 and 1. On a number line, divide the interval between 0 and 1 into 10 equal parts. Now label the parts 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0. We write 0 as 0.0 and 1 and 1.0, so that the numbers are consistently in tenths. Finally, mark 0.4 on the number line. See [link].



Note:
Exercise:

Problem: Locate on the number line: 0.6.

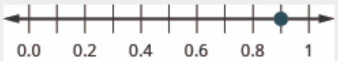
Solution:



Note:
Exercise:

Problem: Locate on the number line: 0.9.

Solution:

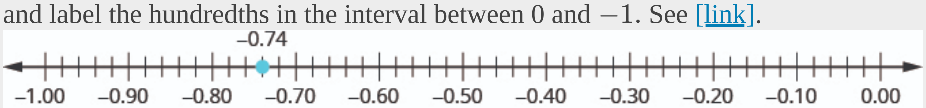


Example:
Exercise:

Problem: Locate -0.74 on the number line.

Solution:

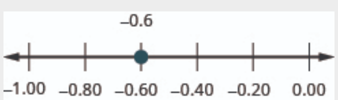
The decimal -0.74 is equivalent to $-\frac{74}{100}$, so it is located between 0 and -1. On a number line, mark off and label the hundredths in the interval between 0 and -1. See [link].



Note:
Exercise:

Problem: Locate on the number line: -0.6.

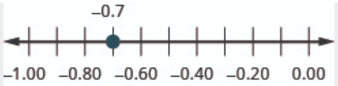
Solution:



Note:
Exercise:

Problem: Locate on the number line: -0.7.

Solution:



Which is larger, 0.04 or 0.40? If you think of this as money, you know that \$0.40 (forty cents) is greater than \$0.04 (four cents). So,

$0.40 > 0.04$

Again, we can use the number line to order numbers.

- $a < b$ "*a* is less than *b*" when *a* is to the left of *b* on the number line
- $a > b$ "*a* is greater than *b*" when *a* is to the right of *b* on the number line

Where are 0.04 and 0.40 located on the number line? See [link].

We see that 0.40 is to the right of 0.04 on the number line. This is another way to demonstrate that 0.40 > 0.04.

How does 0.31 compare to 0.308? This doesn't translate into money to make it easy to compare. But if we convert 0.31 and 0.308 into fractions, we can tell which is larger.

	0.31	0.308
Convert to fractions.	$\frac{31}{100}$	$\frac{308}{1000}$
We need a common denominator to compare them.	$\frac{31 \cdot 10}{100 \cdot 10}$	$\frac{308}{1000}$
	$\frac{310}{1000}$	$\frac{308}{1000}$

Because 310 > 308, we know that $\frac{310}{1000} > \frac{308}{1000}$. Therefore, 0.31 > 0.308.

Notice what we did in converting 0.31 to a fraction—we started with the fraction $\frac{31}{100}$ and ended with the equivalent fraction $\frac{310}{1000}$. Converting $\frac{310}{1000}$ back to a decimal gives 0.310. So 0.31 is equivalent to 0.310. Writing zeros at the end of a decimal does not change its value!
Equation:

$$\frac{31}{100} = \frac{310}{1000} \quad \text{and} \quad 0.31 = 0.310$$

We say 0.31 and 0.310 are **equivalent decimals**.

Note:
Equivalent Decimals
Two decimals are equivalent if they convert to equivalent fractions.

We use equivalent decimals when we order decimals.

The steps we take to order decimals are summarized here.

Note:
Order Decimals.

Write the numbers one under the other, lining up the decimal points.
Check to see if both numbers have the same number of digits. If not, write zeros at the end of the one with fewer digits to make them match.
Compare the numbers as if they were whole numbers.
Order the numbers using the appropriate inequality sign.

Example:
Exercise:

Problem: Order 0.64 ___ 0.6 using $<$ or $>$.

Solution:
Solution

Write the numbers one under the other, lining up the decimal points.	$\begin{array}{l}0.64\\0.6\end{array}$
Add a zero to 0.6 to make it a decimal with 2 decimal places. Now they are both hundredths.	$\begin{array}{l}0.64\\0.60\end{array}$
64 is greater than 60.	$64 > 60$
64 hundredths is greater than 60 hundredths.	$0.64 > 0.60$
	$0.64 > 0.6$

Note:
Exercise:

Problem: Order each of the following pairs of numbers, using $<$ or $>$: 0.42 ___ 0.4.

Solution:

$>$

Note:
Exercise:

Problem: Order each of the following pairs of numbers, using $<$ or $>$: 0.18 ___ 0.1.

Solution:

$>$

Example:
Exercise:

Problem: Order 0.83 ___ 0.803 using $<$ or $>$.

Solution:
Solution

	0.83 ___ 0.803
Write the numbers one under the other, lining up the decimals.	0.83 0.803
They do not have the same number of digits. Write one zero at the end of 0.83.	0.830 0.803
Since $830 > 803$, 830 thousandths is greater than 803 thousandths.	$0.830 > 0.803$
	$0.83 > 0.803$

Note:
Exercise:

Problem: Order the following pair of numbers, using $<$ or $>$: 0.76 ___ 0.706.

Solution:

$>$

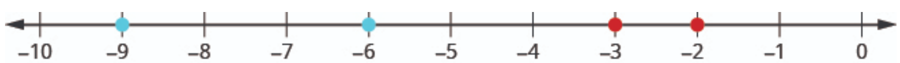
Note:
Exercise:

Problem: Order the following pair of numbers, using $<$ or $>$: 0.305 ___ 0.35.

Solution:

$<$

When we order negative decimals, it is important to remember how to order negative integers. Recall that larger numbers are to the right on the number line. For example, because -2 lies to the right of -3 on the number line, we know that $-2 > -3$. Similarly, smaller numbers lie to the left on the number line. For example, because -9 lies to the left of -6 on the number line, we know that $-9 < -6$. See [link].



If we zoomed in on the interval between 0 and -1, as shown in [link], we would see in the same way that $-0.2 > -0.3$ and $-0.9 < -0.6$.

Example:
Exercise:

Problem: Use $<$ or $>$ to order -0.1 ___ -0.8.

Solution:
Solution

	-0.1 ___ -0.8
Write the numbers one under the other, lining up the decimal points. They have the same number of digits.	-0.1 -0.8
Since $-1 > -8$, -1 tenth is greater than -8 tenths.	$-0.1 > -0.8$

Note:
Exercise:

Problem: Order the following pair of numbers, using < or >: -0.3 ___ -0.5.

Solution:

$>$

Note:
Exercise:

Problem: Order the following pair of numbers, using < or >: -0.6 ___ -0.7.

Solution:

>

Key Concepts

- **Square Root Notation**
  $\sqrt{m}$ is read 'the square root of m.' If $m = n^2$, then $\sqrt{m} = n$, for $n \geq 0$.
- **Order Decimals**

  Write the numbers one under the other, lining up the decimal points.
  Check to see if both numbers have the same number of digits. If not, write zeros at the end of the one with fewer digits to make them match.
  Compare the numbers as if they were whole numbers.
  Order the numbers using the appropriate inequality sign.

Practice Makes Perfect

Simplify Expressions with Square Roots

In the following exercises, simplify.
Exercise:

Problem: $\sqrt{36}$

Solution:

6

Exercise:

Problem: $\sqrt{4}$

Exercise:

Problem: $\sqrt{64}$

Solution:

8

Exercise:

Problem: $\sqrt{169}$

Exercise:

Problem: $\sqrt{9}$

Solution:

3

Exercise:

Problem: $\sqrt{16}$

Exercise:

Problem: $\sqrt{100}$

Solution:

10

Exercise:

Problem: $\sqrt{144}$

Exercise:

Problem: $-\sqrt{4}$

Solution:

-2

Exercise:

Problem: $-\sqrt{100}$

Exercise:

Problem: $-\sqrt{1}$

Solution:

-1

Exercise:

Problem: $-\sqrt{121}$

Identify Integers, Rational Numbers, Irrational Numbers, and Real Numbers

In the following exercises, write as the ratio of two integers.

Exercise:

Problem: ⓐ 5 ⓑ 3.19

Solution:

ⓐ $\frac{5}{1}$ ⓑ $\frac{319}{100}$

Exercise:

Problem: ⓐ 8 ⓑ 1.61

Exercise:

Problem: ⓐ -12 ⓑ 9.279

Solution:

ⓐ $\frac{-12}{1}$ ⓑ $\frac{9297}{1000}$

Exercise:

Problem: ⓐ -16 ⓑ 4.399

In the following exercises, list the ⓐ rational numbers, ⓑ irrational numbers

Exercise:

Problem: $0.75, 0.22\overline{3}, 1.39174$

Solution:

ⓐ $0.75, 0.22\overline{3}$ ⓑ $1.39174...$

Exercise:

Problem: $0.36, 0.94729..., 2.52\overline{8}$

Exercise:

Problem: $0.4\overline{5}, 1.919293..., 3.59$

Solution:

ⓐ $0.4\overline{5}, 3.59$ ⓑ $1.919293...$

Exercise:

Problem: $0.1\overline{3}, 0.42982..., 1.875$

In the following exercises, identify whether each number is rational or irrational.

Exercise:

Problem: ⓐ $\sqrt{25}$ ⓑ $\sqrt{30}$

Solution:

ⓐ rational ⓑ irrational

Exercise:

Problem: ⓐ $\sqrt{44}$ ⓑ $\sqrt{49}$

Exercise:

Problem: ⓐ $\sqrt{164}$ ⓑ $\sqrt{169}$

Solution:

ⓐ irrational ⓑ rational

Exercise:

Problem: ⓐ $\sqrt{225}$ ⓑ $\sqrt{216}$

In the following exercises, identify whether each number is a real number or not a real number.

Exercise:

Problem: ⓐ $-\sqrt{81}$ ⓑ $\sqrt{-121}$

Solution:

ⓐ real number ⓑ not a real number

Exercise:

Problem: ⓐ $-\sqrt{64}$ ⓑ $\sqrt{-9}$

Exercise:

Problem: ⓐ $\sqrt{-36}$ ⓑ $-\sqrt{144}$

Solution:

ⓐ not a real number ⓑ real number

Exercise:

Problem: ⓐ $\sqrt{-49}$ ⓑ $-\sqrt{144}$

In the following exercises, list the ⓐ whole numbers, ⓑ integers, ⓒ rational numbers, ⓓ irrational numbers, ⓔ real numbers for each set of numbers.

Exercise:

Problem: $-8, 0, 1.95286..., \frac{12}{5}, \sqrt{36}, 9$

Solution:

ⓐ $0, \sqrt{36}, 9$ ⓑ $-8, \sqrt{36}, 9$ ⓒ $-8, 0, \frac{12}{5}, \sqrt{36}, 9$ ⓓ $1.95286...$ ⓔ $-8, 0, 1.95286..., \frac{12}{5}, \sqrt{36}, 9$

Exercise:

Problem: $-9, -3\frac{4}{9}, -\sqrt{9}, 0.40\overline{9}, \frac{11}{6}, 7$

Exercise:

Problem: $-\sqrt{100}, -7, -\frac{8}{3}, -1, 0.77, 3\frac{1}{4}$

Solution:

ⓐ none ⓑ $-\sqrt{100}, -7, -1$ ⓒ $-\sqrt{100}, -7, -\frac{8}{3}, -1, 0.77, 3\frac{1}{4}$ ⓓ none ⓔ $-\sqrt{100}, -7, -\frac{8}{3}, -1, 0.77, 3\frac{1}{4}$

Exercise:

Problem: $-6, -\frac{5}{2}, 0, 0.7\overline{1428}5, 2\frac{1}{5}, \sqrt{14}$

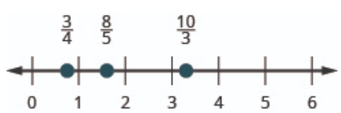
Locate Fractions on the Number Line

In the following exercises, locate the numbers on a number line.

Exercise:

Problem: $\frac{3}{4}, \frac{8}{5}, \frac{10}{3}$

Solution:



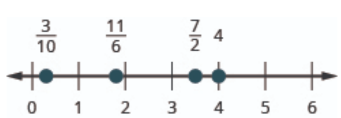
Exercise:

Problem: $\frac{1}{4}, \frac{9}{5}, \frac{11}{3}$

Exercise:

Problem: $\frac{3}{10}, \frac{7}{2}, \frac{11}{6}, 4$

Solution:



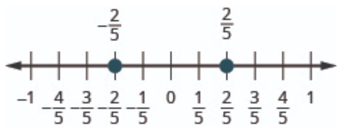
Exercise:

Problem: $\frac{7}{10}, \frac{5}{2}, \frac{13}{8}, 3$

Exercise:

Problem: $\frac{2}{5}, -\frac{2}{5}$

Solution:



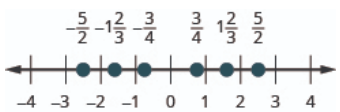
Exercise:

Problem: $\frac{3}{4}, -\frac{3}{4}$

Exercise:

Problem: $\frac{3}{4}, -\frac{3}{4}, 1\frac{2}{3}, -1\frac{2}{3}, \frac{5}{2}, -\frac{5}{2}$

Solution:



Exercise:

Problem: $\frac{1}{5}, -\frac{2}{5}, 1\frac{3}{4}, -1\frac{3}{4}, \frac{8}{3}, -\frac{8}{3}$

In the following exercises, order each of the pairs of numbers, using < or >.

Exercise:

Problem: -1 ____ $-\frac{1}{4}$

Solution:

$<$

Exercise:

Problem: -1 ____ $-\frac{1}{3}$

Exercise:

Problem: $-2\frac{1}{2}$ ____ -3

Solution:

$>$

Exercise:

Problem: $-1\frac{3}{4}$ ____ -2

Exercise:

Problem: $-\frac{5}{12}$ ____ $-\frac{7}{12}$

Solution:

$>$

Exercise:

Problem: $-\frac{9}{10}$ ___ $-\frac{3}{10}$

Exercise:

Problem: -3 ___ $-\frac{13}{5}$

Solution:

$<$

Exercise:

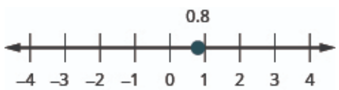
Problem: -4 ___ $-\frac{23}{6}$

Locate Decimals on the Number Line In the following exercises, locate the number on the number line.

Exercise:

Problem: 0.8

Solution:



Exercise:

Problem: -0.9

Exercise:

Problem: -1.6

Solution:

Exercise:

Problem: 3.1

In the following exercises, order each pair of numbers, using < or >.

Exercise:

Problem: 0.37 ___ 0.63

Solution:

$<$

Exercise:

Problem: 0.86 ___ 0.69

Exercise:

Problem: 0.91 ___ 0.901

Solution:

$>$

Exercise:

Problem: 0.415 ___ 0.41

Exercise:

Problem: -0.5 ___ -0.3

Solution:

$<$

Exercise:

Problem: -0.1 ___ -0.4

Exercise:

Problem: -0.62 ___ -0.619

Solution:

$<$

Exercise:

Problem: -7.31 ___ -7.3

Everyday Math

Exercise:

Problem:

Field trip All the 5th graders at Lincoln Elementary School will go on a field trip to the science museum. Counting all the children, teachers, and chaperones, there will be 147 people. Each bus holds 44 people.

ⓐ How many busses will be needed?
ⓑ Why must the answer be a whole number?
ⓒ Why shouldn't you round the answer the usual way, by choosing the whole number closest to the exact answer?

Solution:

ⓐ 4 busses ⓑ answers may vary ⓒ answers may vary

Exercise:

Problem:

Child care Serena wants to open a licensed child care center. Her state requires there be no more than 12 children for each teacher. She would like her child care center to serve 40 children.

ⓐ How many teachers will be needed?
ⓑ Why must the answer be a whole number?
ⓒ Why shouldn't you round the answer the usual way, by choosing the whole number closest to the exact answer?

Writing Exercises

Exercise:

Problem: In your own words, explain the difference between a rational number and an irrational number.

Solution:

Answers may vary

Exercise:

Problem:

Explain how the sets of numbers (counting, whole, integer, rational, irrationals, reals) are related to each other.

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objective of this section.

I can...	Confidently	With some help	No-I don't get it!
simplify expressions with square roots.			
identify integers, rational numbers, irrational numbers, and real numbers.			
locate fractions on the number line.			
locate decimals on the number line.			

ⓑ On a scale of $1-10$, how would you rate your mastery of this section in light of your responses on the checklist? How can you improve this?

Glossary

equivalent decimals
: Two decimals are equivalent if they convert to equivalent fractions.

irrational number

An irrational number is a number that cannot be written as the ratio of two integers. Its decimal form does not stop and does not repeat.

rational number

A rational number is a number of the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$. A rational number can be written as the ratio of two integers. Its decimal form stops or repeats.

radical sign

A radical sign is the symbol $\sqrt{m}$ that denotes the positive square root.

real number

A real number is a number that is either rational or irrational.

square and square root

If $n^2 = m$, then m is the square of n and n is a square root of m.

Properties of Real Numbers
By the end of this section, you will be able to:

- Use the commutative and associative properties
- Use the identity and inverse properties of addition and multiplication
- Use the properties of zero
- Simplify expressions using the distributive property

Note:
A more thorough introduction to the topics covered in this section can be found in the *Prealgebra* chapter, **The Properties of Real Numbers**.

Use the Commutative and Associative Properties

Think about adding two numbers, say 5 and 3. The order we add them doesn't affect the result, does it?
Equation:

$$\begin{array}{cc} 5+3 & 3+5 \\ 8 & 8 \end{array}$$

Equation:

$$5+3=3+5$$

The results are the same.

As we can see, the order in which we add does not matter!

What about multiplying $5\text{ and }3$?
Equation:

$$\begin{array}{cc} 5\cdot 3 & 3\cdot 5 \\ 15 & 15 \end{array}$$

Equation:

$$5\cdot 3=3\cdot 5$$

Again, the results are the same!

The order in which we multiply does not matter!

These examples illustrate the **commutative property**. When adding or multiplying, changing the *order* gives the same result.

Note:
Commutative Property
Equation:

$$\begin{array}{llrcl} \textbf{of Addition} & \text{If } a, b \text{ are real numbers, then} & a+b & = & b+a \\ \textbf{of Multiplication} & \text{If } a, b \text{ are real numbers, then} & a \cdot b & = & b \cdot a \end{array}$$

When adding or multiplying, changing the *order* gives the same result.

The commutative property has to do with order. If you change the order of the numbers when adding or multiplying, the result is the same.

What about subtraction? Does order matter when we subtract numbers? Does $7-3$ give the same result as $3-7$?
Equation:

$$\begin{array}{cc} 7-3 & 3-7 \\ 4 & -4 \end{array}$$

$$4 \neq -4$$
$$7-3 \neq 3-7$$

The results are not the same.

Since changing the order of the subtraction did not give the same result, we know that *subtraction is not commutative*.

Let's see what happens when we divide two numbers. Is division commutative?
Equation:

$$\begin{array}{cc} 12 \div 4 & 4 \div 12 \\ \frac{12}{4} & \frac{4}{12} \\ 3 & \frac{1}{3} \end{array}$$

$$3 \neq \frac{1}{3}$$
$$12 \div 4 \neq 4 \div 12$$

The results are not the same.

Since changing the order of the division did not give the same result, *division is not commutative*. The commutative properties only apply to addition and multiplication!

- Addition and multiplication *are* commutative.
- Subtraction and Division *are not* commutative.

If you were asked to simplify this expression, how would you do it and what would your answer be?
Equation:

$$7+8+2$$

Some people would think $7+8$ is 15 and then $15+2$ is 17. Others might start with $8+2$ makes 10 and then $7+10$ makes 17.

Either way gives the same result. Remember, we use parentheses as grouping symbols to indicate which operation should be done first.
Equation:

$$\begin{array}{ll} & (7+8)+2 \\ \text{Add } 7+8. & 15+2 \\ \text{Add.} & 17 \end{array}$$

$$\begin{array}{ll} & 7+(8+2) \\ \text{Add } 8+2. & 7+10 \\ \text{Add.} & 17 \end{array}$$

$$(7+8)+2=7+(8+2)$$

When adding three numbers, changing the grouping of the numbers gives the same result.

This is true for multiplication, too.
Equation:

$$\begin{array}{ll} & \left(5\cdot\frac{1}{3}\right)\cdot 3 \\ \text{Multiply.}\quad 5\cdot\frac{1}{3} & \frac{5}{3}\cdot 3 \\ \text{Multiply.} & 5 \end{array}$$

$$\begin{array}{ll} & 5\cdot\left(\frac{1}{3}\cdot 3\right) \\ \text{Multiply.}\quad \frac{1}{3}\cdot 3 & 5\cdot 1 \\ \text{Multiply.} & 5 \end{array}$$

$$\left(5\cdot\frac{1}{3}\right)\cdot 3=5\cdot\left(\frac{1}{3}\cdot 3\right)$$

When multiplying three numbers, changing the grouping of the numbers gives the same result.

You probably know this, but the terminology may be new to you. These examples illustrate the **associative property**.

Note:
Associative Property
Equation:

of Addition	If a, b, c are real numbers, then $(a+b)+c=a+(b+c)$
of Multiplication	If a, b, c are real numbers, then $(a\cdot b)\cdot c=a\cdot(b\cdot c)$

When adding or multiplying, changing the *grouping* gives the same result.

Let's think again about multiplying $5\cdot\frac{1}{3}\cdot 3$. We got the same result both ways, but which way was easier? Multiplying $\frac{1}{3}$ and 3 first, as shown above on the right side, eliminates the fraction in the first step. Using the associative property can make the math easier!

The associative property has to do with grouping. If we change how the numbers are grouped, the result will be the same. Notice it is the same three numbers in the same order—the only difference is the grouping.

We saw that subtraction and division were not commutative. They are not associative either.

When simplifying an expression, it is always a good idea to plan what the steps will be. In order to combine like terms in the next example, we will use the commutative property of addition to write the like terms together.

Example:
Exercise:

Problem: Simplify: $18p+6q+15p+5q$.

Solution:
Solution

	$18p+6q+15p+5q$
Use the commutative property of addition to re-order so that like terms are together.	$18p+15p+6q+5q$
Add like terms.	$33p+11q$

Note:
Exercise:

Problem: Simplify: $23r+14s+9r+15s$.

Solution:

$32r+29s$

Note:
Exercise:

Problem: Simplify: $37m+21n+4m-15n$.

Solution:

$41m+6n$

When we have to simplify algebraic expressions, we can often make the work easier by applying the commutative or associative property first, instead of automatically following the order of operations. When adding or subtracting fractions, combine those with a common denominator first.

Example:
Exercise:

Problem: Simplify: $\left(\frac{5}{13}+\frac{3}{4}\right)+\frac{1}{4}$.

Solution:
Solution

	$\left(\frac{5}{13}+\frac{3}{4}\right)+\frac{1}{4}$
Notice that the last 2 terms have a common denominator, so change the grouping.	$\frac{5}{13}+\left(\frac{3}{4}+\frac{1}{4}\right)$
Add in parentheses first.	$\frac{5}{13}+\left(\frac{4}{4}\right)$
Simplify the fraction.	$\frac{5}{13}+1$
Add.	$1\frac{5}{13}$
Convert to an improper fraction.	$\frac{18}{13}$

Note:
Exercise:

Problem: Simplify: $\left(\frac{7}{15}+\frac{5}{8}\right)+\frac{3}{8}$.

Solution:

$1\,\frac{7}{15}$

Note:
Exercise:

Problem: Simplify: $\left(\frac{2}{9}+\frac{7}{12}\right)+\frac{5}{12}$.

Solution:

$1\,\frac{2}{9}$

Example:
Exercise:

Problem: Use the associative property to simplify $6\,(3x)$.

Solution:
Solution

Use the associative property of multiplication, $(a\cdot b)\cdot c=a\cdot(b\cdot c)$, to change the grouping.

	$6\,(3x)$
Change the grouping.	$(6\cdot 3)x$
Multiply in the parentheses.	$18x$

Notice that we can multiply $6\cdot 3$ but we could not multiply $3x$ without having a value for x.

Note:
Exercise:

Problem: Use the associative property to simplify $8(4x)$.

Solution:

$32x$

Note:
Exercise:

Problem: Use the associative property to simplify $-9\,(7y)$.

Solution:

$-63y$

Use the Identity and Inverse Properties of Addition and Multiplication

What happens when we add 0 to any number? Adding 0 doesn't change the value. For this reason, we call 0 the **additive identity**.

For example,
Equation:

$$\begin{array}{ccc} 13+0 & \quad -14+0 & \quad 0+(-8) \\ 13 & \quad -14 & \quad -8 \end{array}$$

These examples illustrate the **Identity Property of Addition** that states that for any real number a, $a+0=a$ and $0+a=a$.

What happens when we multiply any number by one? Multiplying by 1 doesn't change the value. So we call 1 the **multiplicative identity**.

For example,
Equation:

$$\begin{array}{ccc} 43\cdot 1 & \quad -27\cdot 1 & \quad 1\cdot \frac{3}{5} \\ 43 & \quad -27 & \quad \frac{3}{5} \end{array}$$

These examples illustrate the **Identity Property of Multiplication** that states that for any real number a, $a\cdot 1=a$ and $1\cdot a=a$.

We summarize the Identity Properties below.

Note:
Identity Property
Equation:

of addition For any real number a: $a+0=a$ $0+a=a$
0 is the **additive identity**

of multiplication For any real number a: $a \cdot 1=a$ $1 \cdot a=a$
1 is the **multiplicative identity**

What number added to 5 gives the additive identity, 0?
$5 + \_\_\_\_\_ = 0$ We know $5 + (-5) = 0$
What number added to −6 gives the additive identity, 0?
$-6 + \_\_\_\_\_ = 0$ We know $-6 + 6 = 0$

Notice that in each case, the missing number was the opposite of the number!

We call $-a$. the **additive inverse** of a. *The opposite of a number is its additive inverse.* A number and its opposite add to zero, which is the additive identity. This leads to the **Inverse Property of Addition** that states for any real number $a, a+(-a)=0$. Remember, a number and its opposite add to zero.

What number multiplied by $\frac{2}{3}$ gives the multiplicative identity, 1? In other words, $\frac{2}{3}$ times what results in 1?

$\frac{2}{3} \cdot \_\_\_\_ = 1$ We know $\frac{2}{3} \cdot \frac{3}{2} = 1$

What number multiplied by 2 gives the multiplicative identity, 1? In other words 2 times what results in 1?

$2 \cdot \_\_\_\_ = 1$ We know $2 \cdot \frac{1}{2} = 1$

Notice that in each case, the missing number was the reciprocal of the number!

We call $\frac{1}{a}$ the **multiplicative inverse** of a. *The reciprocal of a number is its multiplicative inverse.* A number and its reciprocal multiply to one, which is the multiplicative identity. This leads to the **Inverse Property of Multiplication** that states that for any real number $a, a \neq 0, a \cdot \frac{1}{a}=1$.

We'll formally state the inverse properties here:

Note:
Inverse Property
Equation:

of addition	For any real number a, $-a.$ is the **additive inverse** of a. A number and its opposite add to zero.	$a+(-a)=0$
of multiplication	For any real number $a, a \neq 0$ $\frac{1}{a}$ is the **multiplicative inverse** of a. A number and its reciprocal multiply to one.	$a \cdot \frac{1}{a}=1$

Example:
Exercise:

Problem: Find the additive inverse of ⓐ $\frac{5}{8}$ ⓑ 0.6 ⓒ -8 ⓓ $-\frac{4}{3}$.

Solution:
Solution

To find the additive inverse, we find the opposite.

ⓐ The additive inverse of $\frac{5}{8}$ is the opposite of $\frac{5}{8}$. The additive inverse of $\frac{5}{8}$ is $-\frac{5}{8}$.

ⓑ The additive inverse of 0.6 is the opposite of 0.6. The additive inverse of 0.6 is -0.6.

ⓒ The additive inverse of -8 is the opposite of -8. We write the opposite of -8 as $-(-8)$, and then simplify it to 8. Therefore, the additive inverse of -8 is 8.

ⓓ The additive inverse of $-\frac{4}{3}$ is the opposite of $-\frac{4}{3}$. We write this as $-\left(-\frac{4}{3}\right)$, and then simplify to $\frac{4}{3}$. Thus, the additive inverse of $-\frac{4}{3}$ is $\frac{4}{3}$.

Note:
Exercise:

Problem: Find the additive inverse of: ⓐ $\frac{7}{9}$ ⓑ 1.2 ⓒ -14 ⓓ $-\frac{9}{4}$.

Solution:

ⓐ $-\frac{7}{9}$ ⓑ -1.2 ⓒ 14 ⓓ $\frac{9}{4}$

Note:
Exercise:

Problem: Find the additive inverse of: ⓐ $\frac{7}{13}$ ⓑ 8.4 ⓒ -46 ⓓ $-\frac{5}{2}$.

Solution:

ⓐ $-\frac{7}{13}$ ⓑ -8.4 ⓒ 46 ⓓ $\frac{5}{2}$

Example:
Exercise:

Problem: Find the multiplicative inverse of ⓐ 9 ⓑ $-\frac{1}{9}$ ⓒ 0.9.

Solution:
Solution

To find the multiplicative inverse, we find the reciprocal.

ⓐ The multiplicative inverse of 9 is the reciprocal of 9, which is $\frac{1}{9}$. Therefore, the multiplicative inverse of 9 is $\frac{1}{9}$.
ⓑ The multiplicative inverse of $-\frac{1}{9}$ is the reciprocal of $-\frac{1}{9}$, which is -9. Thus, the multiplicative inverse of $-\frac{1}{9}$ is -9.
ⓒ To find the multiplicative inverse of 0.9, we first convert 0.9 to a fraction, $\frac{9}{10}$. Then we find the reciprocal of the fraction. The reciprocal of $\frac{9}{10}$ is $\frac{10}{9}$. So the multiplicative inverse of 0.9 is $\frac{10}{9}$.

Note:
Exercise:

Problem: Find the multiplicative inverse of ⓐ 4 ⓑ $-\frac{1}{7}$ ⓒ 0.3

Solution:

ⓐ $\frac{1}{4}$ ⓑ -7 ⓒ $\frac{10}{3}$

Note:
Exercise:

Problem: Find the multiplicative inverse of ⓐ 18 ⓑ $-\frac{4}{5}$ ⓒ 0.6.

Solution:

ⓐ $\frac{1}{18}$ ⓑ $-\frac{5}{4}$ ⓒ $\frac{5}{3}$

Use the Properties of Zero

The identity property of addition says that when we add 0 to any number, the result is that same number. What happens when we multiply a number by 0? Multiplying by 0 makes the product equal zero.

Note:
Multiplication by Zero
For any real number a.
Equation:

$$a \cdot 0 = 0 \qquad\qquad 0 \cdot a = 0$$

The product of any real number and 0 is 0.

What about division involving zero? What is $0 \div 3$? Think about a real example: If there are no cookies in the cookie jar and 3 people are to share them, how many cookies does each person get? There are no cookies to share, so each person gets 0 cookies. So,
Equation:

$$0 \div 3 = 0$$

We can check division with the related multiplication fact.
Equation:

$$12 \div 6 = 2 \text{ because } 2 \cdot 6 = 12.$$

So we know $0 \div 3 = 0$ because $0 \cdot 3 = 0$.

Note:
Division of Zero
For any real number a, except 0, $\frac{0}{a} = 0$ and $0 \div a = 0$.
Zero divided by any real number except zero is zero.

Now think about dividing *by* zero. What is the result of dividing 4 by 0? Think about the related multiplication fact: $4 \div 0 = ?$ means $? \cdot 0 = 4$. Is there a number that multiplied by 0 gives 4? Since

any real number multiplied by 0 gives 0, there is no real number that can be multiplied by 0 to obtain 4.

We conclude that there is no answer to $4 \div 0$ and so we say that division by 0 is undefined.

Note:
Division by Zero
For any real number a, except 0, $\frac{a}{0}$ and $a \div 0$ are undefined.
Division by zero is undefined.

We summarize the properties of zero below.

Note:
Properties of Zero
Multiplication by Zero: For any real number a,

$a \cdot 0 = 0 \quad 0 \cdot a = 0$ The product of any number and 0 is 0.

Division of Zero, Division by Zero: For any real number $a, a \neq 0$

$\frac{0}{a} = 0$	Zero divided by any real number, except itself is zero.
$\frac{a}{0}$ is undefined	Division by zero is undefined.

Example:
Exercise:

Problem: Simplify: ⓐ $-8 \cdot 0$ ⓑ $\frac{0}{-2}$ ⓒ $\frac{-32}{0}$.

Solution:
Solution

ⓐ

	$-8 \cdot 0$
The product of any real number and 0 is 0.	0

ⓑ

	$\frac{0}{-2}$
Zero divided by any real number, except itself, is 0.	0

ⓒ

	$\frac{-32}{0}$
Division by 0 is undefined.	Undefined

Note:
Exercise:

Problem: Simplify: ⓐ $-14 \cdot 0$ ⓑ $\frac{0}{-6}$ ⓒ $\frac{-2}{0}$.

Solution:

ⓐ 0 ⓑ 0 ⓒ undefined

Note:
Exercise:

Problem: Simplify: ⓐ $0\left(-17\right)$ ⓑ $\frac{0}{-10}$ ⓒ $\frac{-5}{0}$.

Solution:

ⓐ 0 ⓑ 0 ⓒ undefined

We will now practice using the properties of identities, inverses, and zero to simplify expressions.

Example:
Exercise:

Problem: Simplify: ⓐ $\frac{0}{n+5}$, where $n \ne -5$ ⓑ $\frac{10-3p}{0}$, where $10-3p \ne 0$.

Solution:
Solution

ⓐ

	$\frac{0}{n+5}$
Zero divided by any real number except itself is 0.	0

ⓑ

	$\frac{10-3p}{0}$
Division by 0 is undefined.	Undefined

Example:
Exercise:

Problem: Simplify: $-84n + (-73n) + 84n$.

Solution:
Solution

	$-84n + (-73n) + 84n$
Notice that the first and third terms are opposites; use the commutative property of addition to re-order the terms.	$-84n + 84n + (-73n)$
Add left to right.	$0 + (-73)$
Add.	$-73n$

Note:
Exercise:

Problem: Simplify: $-27a + (-48a) + 27a$.

Solution:

$-48a$

Note:
Exercise:

Problem: Simplify: $39x + (-92x) + (-39x)$.

Solution:

$-92x$

Now we will see how recognizing reciprocals is helpful. Before multiplying left to right, look for reciprocals—their product is 1.

Example:
Exercise:

Problem: Simplify: $\frac{7}{15} \cdot \frac{8}{23} \cdot \frac{15}{7}$.

Solution:
Solution

	$\frac{7}{15} \cdot \frac{8}{23} \cdot \frac{15}{7}$
Notice the first and third terms are reciprocals, so use the commutative property of multiplication to re-order the factors.	$\frac{7}{15} \cdot \frac{15}{7} \cdot \frac{8}{23}$
Multiply left to right.	$1 \cdot \frac{8}{23}$
Multiply.	$\frac{8}{23}$

Note:
Exercise:

Problem: Simplify: $\frac{9}{16} \cdot \frac{5}{49} \cdot \frac{16}{9}$.

Solution:

$\frac{5}{49}$

Note:
Exercise:

Problem: Simplify: $\frac{6}{17} \cdot \frac{11}{25} \cdot \frac{17}{6}$.

Solution:

$\frac{11}{25}$

Note:
Exercise:

Problem: Simplify: ⓐ $\frac{0}{m+7}$, where $m \neq -7$ ⓑ $\frac{18-6c}{0}$, where $18-6c \neq 0$.

Solution:

ⓐ 0 ⓑ undefined

Note:
Exercise:

Problem: Simplify: ⓐ $\frac{0}{d-4}$, where $d \neq 4$ ⓑ $\frac{15-4q}{0}$, where $15-4q \neq 0$.

Solution:

ⓐ 0 ⓑ undefined

Example:
Exercise:

Problem: Simplify: $\frac{3}{4} \cdot \frac{4}{3}(6x+12)$.

Solution:
Solution

	$\frac{3}{4} \cdot \frac{4}{3}(6x+12)$
There is nothing to do in the parentheses, so multiply the two fractions first—notice, they are reciprocals.	$1\,(6x+12)$
Simplify by recognizing the multiplicative identity.	$6x+12$

Note:
Exercise:

Problem: Simplify: $\frac{2}{5} \cdot \frac{5}{2}(20y+50)$.

Solution:

$20y+50$

Note:
Exercise:

Problem: Simplify: $\frac{3}{8}\cdot\frac{8}{3}(12z+16)$.

Solution:

$12z+16$

Simplify Expressions Using the Distributive Property

Suppose that three friends are going to the movies. They each need $9.25—that's 9 dollars and 1 quarter—to pay for their tickets. How much money do they need all together?

You can think about the dollars separately from the quarters. They need 3 times $9 so $27, and 3 times 1 quarter, so 75 cents. In total, they need $27.75. If you think about doing the math in this way, you are using the **distributive property**.

Note:
Distributive Property
Equation:

$$\begin{array}{rl}\text{If } a,b,c \text{ are real numbers, then} & a(b+c)=ab+ac \\ \text{Also,} & (b+c)a=ba+ca \\ & a(b-c)=ab-ac \\ & (b-c)a=ba-ca\end{array}$$

Back to our friends at the movies, we could find the total amount of money they need like this:
Equation:

$$\begin{array}{rcl} & 3(9.25) & \\ 3(9 & + & 0.25) \\ 3(9) & + & 3(0.25) \\ 27 & + & 0.75 \\ & 27.75 & \end{array}$$

In algebra, we use the **distributive property** to remove parentheses as we simplify expressions.

For example, if we are asked to simplify the expression $3\left(x+4\right)$, the order of operations says to work in the parentheses first. But we cannot add *x* and 4, since they are not like terms. So we use the distributive property, as shown in [link].

Example:
Exercise:

Problem: Simplify: $3\left(x+4\right)$.

Solution:
Solution

	$3\left(x+4\right)$
Distribute.	$3\cdot x+3\cdot 4$
Multiply.	$3x+12$

Note:
Exercise:

Problem: Simplify: $4\left(x+2\right)$.

Solution:

$4x+8$

Note:
Exercise:

Problem: Simplify: $6\left(x+7\right)$.

Solution:

$6x+42$

Some students find it helpful to draw in arrows to remind them how to use the distributive property. Then the first step in [link] would look like this:

$3(x+4)$

Example:
Exercise:

Problem: Simplify: $8\left(\frac{3}{8}x+\frac{1}{4}\right)$.

Solution:
Solution

	$8\left(\frac{3}{8}x+\frac{1}{4}\right)$
Distribute.	$8\cdot\frac{3}{8}x+8\cdot\frac{1}{4}$
Multiply.	$3x+2$

Note:
Exercise:

Problem: Simplify: $6\left(\frac{5}{6}y+\frac{1}{2}\right)$.

Solution:

$5y+3$

Note:
Exercise:

Problem: Simplify: $12\left(\frac{1}{3}n + \frac{3}{4}\right)$.

Solution:

$4n + 9$

Using the distributive property as shown in [link] will be very useful when we solve money applications in later chapters.

Example:
Exercise:

Problem: Simplify: $100\left(0.3 + 0.25q\right)$.

Solution:
Solution

	$100(0.3 + 0.25q)$
Distribute.	$100(0.3) + 100(0.25q)$
Multiply.	$30 + 25q$

Note:
Exercise:

Problem: Simplify: $100\left(0.7 + 0.15p\right)$.

Solution:

$70+15p$

Note:
Exercise:

Problem: Simplify: $100\left(0.04+0.35d\right)$.

Solution:

$4+35d$

When we distribute a negative number, we need to be extra careful to get the signs correct!

Example:
Exercise:

Problem: Simplify: $-2\left(4y+1\right)$.

Solution:
Solution

	$-2(4y+1)$
Distribute.	$-2\cdot 4y+(-2)\cdot 1$
Multiply.	$-8y-2$

Note:
Exercise:

Problem: Simplify: $-3\,(6m+5)$.

Solution:

$-18m-15$

Note:
Exercise:

Problem: Simplify: $-6\,(8n+11)$.

Solution:

$-48n-66$

Example:
Exercise:

Problem: Simplify: $-11\,(4-3a)$.

Solution:
Solution

Distribute.	$-11(4-3a)$
Multiply.	$-11\cdot 4-(-11)\cdot 3a$ $-44-(-33a)$
Simplify.	$-44+33a$

Notice that you could also write the result as $33a-44$. Do you know why?

Note:
Exercise:

Problem: Simplify: $-5\left(2-3a\right)$.

Solution:

$-10+15a$

Note:
Exercise:

Problem: Simplify: $-7\left(8-15y\right)$.

Solution:

$-56+105y$

[link] will show how to use the distributive property to find the opposite of an expression.

Example:
Exercise:

Problem: Simplify: $-\left(y+5\right)$.

Solution:
Solution

	$-\left(y+5\right)$
Multiplying by -1 results in the opposite.	$-1\left(y+5\right)$
Distribute.	$-1\cdot y+\left(-1\right)\cdot 5$
Simplify.	$-y+\left(-5\right)$
	$-y-5$

Note:
Exercise:

Problem: Simplify: $-(z-11)$.

Solution:

$-z+11$

Note:
Exercise:

Problem: Simplify: $-(x-4)$.

Solution:

$-x+4$

There will be times when we’ll need to use the distributive property as part of the order of operations. Start by looking at the parentheses. If the expression inside the parentheses cannot be simplified, the next step would be multiply using the distributive property, which removes the parentheses. The next two examples will illustrate this.

Example:
Exercise:

Problem: Simplify: $8-2(x+3)$.

Be sure to follow the order of operations. Multiplication comes before subtraction, so we will distribute the 2 first and then subtract.

Solution:
Solution

	$8-2(x+3)$
Distribute.	$8-2\cdot x-2\cdot 3$
Multiply.	$8-2x-6$
Combine like terms.	$-2x+2$

Note:
Exercise:

Problem: Simplify: $9 - 3\left(x + 2\right)$.

Solution:

$3 - 3x$

Note:
Exercise:

Problem: Simplify: $7x - 5\left(x + 4\right)$.

Solution:

$2x - 20$

Example:
Exercise:

Problem: Simplify: $4\left(x - 8\right) - \left(x + 3\right)$.

Solution:
Solution

	$4\left(x - 8\right) - \left(x + 3\right)$
Distribute.	$4x - 32 - x - 3$
Combine like terms.	$3x - 35$

Note:
Exercise:

Problem: Simplify: $6\left(x - 9\right) - \left(x + 12\right)$.

Solution:

$5x - 66$

Note:
Exercise:

Problem: Simplify: $8\left(x-1\right)-\left(x+5\right)$.

Solution:

$7x-13$

All the properties of real numbers we have used in this chapter are summarized in [link].

Commutative Property	
of addition If a,b are real numbers, then **of multiplication** If a,b are real numbers, then	$a+b=b+a$ $a\cdot b=b\cdot a$
Associative Property	
of addition If a,b,c are real numbers, then **of multiplication** If a,b,c are real numbers, then	$\left(a+b\right)+c=a+\left(b+c\right)$ $\left(a\cdot b\right)\cdot c=a\cdot\left(b\cdot c\right)$
Distributive Property	
If a,b,c are real numbers, then	$a\left(b+c\right)=ab+ac$
Identity Property	
of addition For any real number a: 0 is the **additive identity** **of multiplication** For any real number a: **1** is the **multiplicative identity**	$a+0=a$ $0+a=a$ $a\cdot 1=a$ $1\cdot a=a$
Inverse Property	
of addition For any real number a, $-a$ is the **additive inverse** of a	$a+\left(-a\right)=0$ $a\cdot\frac{1}{a}=1$

of multiplication For any real number $a, a \neq 0$ $\frac{1}{a}$ is the **multiplicative inverse** of a.	
Properties of Zero	
For any real number a, For any real number $a, a \neq 0$ For any real number $a, a \neq 0$	$a \cdot 0 = 0$ $0 \cdot a = 0$ $\frac{0}{a} = 0$ $\frac{a}{0}$ is undefined

Key Concepts

- **Commutative Property of**
    - **Addition:** If a, b are real numbers, then $a + b = b + a$.
    - **Multiplication:** If a, b are real numbers, then $a \cdot b = b \cdot a$. When adding or multiplying, changing the *order* gives the same result.
- **Associative Property of**
    - **Addition:** If a, b, c are real numbers, then $(a + b) + c = a + (b + c)$.
    - **Multiplication:** If a, b, c are real numbers, then $(a \cdot b) \cdot c = a \cdot (b \cdot c)$. When adding or multiplying, changing the *grouping* gives the same result.
- **Distributive Property:** If a, b, c are real numbers, then
    - $a\,(b + c) = ab + ac$
    - $(b + c)a = ba + ca$
    - $a\,(b - c) = ab - ac$
    - $(b - c)a = ba - ca$
- **Identity Property**
    - **of Addition:** For any real number a: $a + 0 = a \quad 0 + a = a$
      0 is the **additive identity**
    - **of Multiplication:** For any real number a: $a \cdot 1 = a \quad 1 \cdot a = a$
      1 is the **multiplicative identity**
- **Inverse Property**
    - **of Addition:** For any real number $a, a + (-a) = 0$. A number and its *opposite* add to zero.
      $-a$ is the **additive inverse** of a.
    - **of Multiplication:** For any real number $a, (a \neq 0)\ a \cdot \frac{1}{a} = 1$. A number and its *reciprocal* multiply to one. $\frac{1}{a}$ is the **multiplicative inverse** of a.
- **Properties of Zero**

- For any real number a,
  $a \cdot 0 = 0 \;\; 0 \cdot a = 0$ – The product of any real number and 0 is 0.
- $\frac{0}{a} = 0$ for $a \neq 0$ – Zero divided by any real number except zero is zero.
- $\frac{a}{0}$ is undefined – Division by zero is undefined.

Practice Makes Perfect

Use the Commutative and Associative Properties

In the following exercises, use the associative property to simplify.
Exercise:

Problem: $3(4x)$

Solution:

$12x$

Exercise:

Problem: $4(7m)$

Exercise:

Problem: $(y + 12) + 28$

Solution:

$y + 40$

Exercise:

Problem: $(n + 17) + 33$

In the following exercises, simplify.
Exercise:

Problem: $\frac{1}{2} + \frac{7}{8} + \left(-\frac{1}{2}\right)$

Solution:

$\frac{7}{8}$

Exercise:

Problem: $\frac{2}{5} + \frac{5}{12} + \left(-\frac{2}{5}\right)$

Exercise:

Problem: $\frac{3}{20} \cdot \frac{49}{11} \cdot \frac{20}{3}$

Solution:

$\frac{49}{11}$

Exercise:

Problem: $\frac{13}{18} \cdot \frac{25}{7} \cdot \frac{18}{13}$

Exercise:

Problem: $-24.7 \cdot \frac{3}{8}$

Solution:

-63

Exercise:

Problem: $-36 \cdot 11 \cdot \frac{4}{9}$

Exercise:

Problem: $\left(\frac{5}{6} + \frac{8}{15}\right) + \frac{7}{15}$

Solution:

$1\,\frac{5}{6}$

Exercise:

Problem: $\left(\frac{11}{12} + \frac{4}{9}\right) + \frac{5}{9}$

Exercise:

Problem: 17(0.25)(4)

Solution:

17

Exercise:

Problem: 36(0.2)(5)

Exercise:

Problem: [2.48(12)](0.5)

Solution:

14.88

Exercise:

Problem: [9.731(4)](0.75)

Exercise:

Problem: $7(4a)$

Solution:

$28a$

Exercise:

Problem: $9(8w)$

Exercise:

Problem: $-15\left(5m\right)$

Solution:

$-75m$

Exercise:

Problem: $-23\left(2n\right)$

Exercise:

Problem: $12\left(\frac{5}{6}p\right)$

Solution:

$10p$

Exercise:

Problem: $20\left(\frac{3}{5}q\right)$

Exercise:

Problem: $43m+(-12n)+(-16m)+(-9n)$

Solution:

$27m+(-21n)$

Exercise:

Problem: $-22p + 17q + (-35p) + (-27q)$

Exercise:

Problem: $\frac{3}{8}g + \frac{1}{12}h + \frac{7}{8}g + \frac{5}{12}h$

Solution:

$\frac{5}{4}g + \frac{1}{2}h$

Exercise:

Problem: $\frac{5}{6}a + \frac{3}{10}b + \frac{1}{6}a + \frac{9}{10}b$

Exercise:

Problem: $6.8p + 9.14q + (-4.37p) + (-0.88q)$

Solution:

$2.43p + 8.26q$

Exercise:

Problem: $9.6m + 7.22n + (-2.19m) + (-0.65n)$

Use the Identity and Inverse Properties of Addition and Multiplication

In the following exercises, find the additive inverse of each number.

Exercise:

Problem:
ⓐ $\frac{2}{5}$
ⓑ 4.3
ⓒ -8
ⓓ $-\frac{10}{3}$

Solution:

ⓐ $-\frac{2}{5}$ ⓑ -4.3 ⓒ 8 ⓓ $\frac{10}{3}$

Exercise:

ⓐ $\frac{5}{9}$
ⓑ 2.1
ⓒ -3
Problem: ⓓ $-\frac{9}{5}$

Exercise:

ⓐ $-\frac{7}{6}$
ⓑ -0.075
ⓒ 23
Problem: ⓓ $\frac{1}{4}$

Solution:

ⓐ $\frac{7}{6}$ ⓑ 0.075 ⓒ -23 ⓓ $-\frac{1}{4}$

Exercise:

ⓐ $-\frac{8}{3}$
ⓑ -0.019
ⓒ 52
Problem: ⓓ $\frac{5}{6}$

In the following exercises, find the multiplicative inverse of each number.
Exercise:

Problem: ⓐ 6 ⓑ $-\frac{3}{4}$ ⓒ 0.7

Solution:

ⓐ $\frac{1}{6}$ ⓑ $-\frac{4}{3}$ ⓒ $\frac{10}{7}$

Exercise:

Problem: ⓐ 12 ⓑ $-\frac{9}{2}$ ⓒ 0.13

Exercise:

Problem: ⓐ $\frac{11}{12}$ ⓑ -1.1 ⓒ -4

Solution:

ⓐ $\frac{12}{11}$ ⓑ $-\frac{10}{11}$ ⓒ $-\frac{1}{4}$

Exercise:

Problem: ⓐ $\frac{17}{20}$ ⓑ -1.5 ⓒ -3

Use the Properties of Zero

In the following exercises, simplify.

Exercise:

Problem: $\frac{0}{6}$

Solution:

0

Exercise:

Problem: $\frac{3}{0}$

Exercise:

Problem: $0 \div \frac{11}{12}$

Solution:

0

Exercise:

Problem: $\frac{6}{0}$

Exercise:

Problem: $\frac{0}{3}$

Solution:

0

Exercise:

Problem: $0 \cdot \frac{8}{15}$

Exercise:

Problem: $(-3.14)\,(0)$

Solution:

0

Exercise:

Problem: $\frac{\frac{1}{10}}{0}$

Mixed Practice

In the following exercises, simplify.

Exercise:

Problem: $19a + 44 - 19a$

Solution:

44

Exercise:

Problem: $27c + 16 - 27c$

Exercise:

Problem: $10(0.1d)$

Solution:

id

Exercise:

Problem: $100\left(0.01p\right)$

Exercise:

Problem: $\frac{0}{u-4.99}$, where $u \neq 4.99$

Solution:

0

Exercise:

Problem: $\frac{0}{v-65.1}$, where $v \neq 65.1$

Exercise:

Problem: $0 \div \left(x - \frac{1}{2}\right)$, where $x \neq \frac{1}{2}$

Solution:

0

Exercise:

Problem: $0 \div \left(y - \ \frac{1}{6}\right)$, where $x \neq \frac{1}{6}$

Exercise:

Problem: $\frac{32-5a}{0}$, where $32 - 5a \neq 0$

Solution:

undefined

Exercise:

Problem: $\frac{28-9b}{0}$, where $28 - 9b \neq 0$

Exercise:

Problem: $\left(\frac{3}{4} + \frac{9}{10}m\right) \div 0$ where $\frac{3}{4} + \frac{9}{10}m \neq 0$

Solution:

undefined

Exercise:

Problem: $\left(\frac{5}{16}n - \ \frac{3}{7}\right) \div 0$ where $\frac{5}{16}n - \ \frac{3}{7} \neq 0$

Exercise:

Problem: $15 \cdot \frac{3}{5}(4d + 10)$

Solution:

$36d + 90$

Exercise:

Problem: $18 \cdot \frac{5}{6}(15h + 24)$

Simplify Expressions Using the Distributive Property

In the following exercises, simplify using the distributive property.
Exercise:

Problem: $8\left(4y+9\right)$

Solution:

$32y+72$

Exercise:

Problem: $9\left(3w+7\right)$

Exercise:

Problem: $6\left(c-13\right)$

Solution:

$6c-78$

Exercise:

Problem: $7\left(y-13\right)$

Exercise:

Problem: $\frac{1}{4}(3q+12)$

Solution:

$\frac{3}{4}q+3$

Exercise:

Problem: $\frac{1}{5}(4m+20)$

Exercise:

Problem: $9\left(\frac{5}{9}y-\ \frac{1}{3}\right)$

Solution:

$5y-3$

Exercise:

Problem: $10\left(\frac{3}{10}x-\ \frac{2}{5}\right)$

Exercise:

Problem: $12\left(\frac{1}{4}+\frac{2}{3}r\right)$

Solution:

$3 + 8r$

Exercise:

Problem: $12\left(\frac{1}{6} + \frac{3}{4}s\right)$

Exercise:

Problem: $r\left(s - 18\right)$

Solution:

$rs - 18r$

Exercise:

Problem: $u\left(v - 10\right)$

Exercise:

Problem: $(y + 4)p$

Solution:

$yp + 4p$

Exercise:

Problem: $(a + 7)x$

Exercise:

Problem: $-7\left(4p + 1\right)$

Solution:

$-28p - 7$

Exercise:

Problem: $-9\left(9a + 4\right)$

Exercise:

Problem: $-3\left(x - 6\right)$

Solution:

$-3x + 18$

Exercise:

Problem: $-4\left(q-7\right)$

Exercise:

Problem: $-\left(3x-7\right)$

Solution:

$-3x+7$

Exercise:

Problem: $-\left(5p-4\right)$

Exercise:

Problem: $16-3\left(y+8\right)$

Solution:

$-3y-8$

Exercise:

Problem: $18-4\left(x+2\right)$

Exercise:

Problem: $4-11\left(3c-2\right)$

Solution:

$-33c+26$

Exercise:

Problem: $9-6\left(7n-5\right)$

Exercise:

Problem: $22-\left(a+3\right)$

Solution:

$-a+19$

Exercise:

Problem: $8-\left(r-7\right)$

Exercise:

Problem: $(5m-3)-(m+7)$

Solution:

$4m-10$

Exercise:

Problem: $(4y-1)-(y-2)$

Exercise:

Problem: $5\left(2n+9\right)+12\left(n-3\right)$

Solution:

$22n+9$

Exercise:

Problem: $9\left(5u+8\right)+2\left(u-6\right)$

Exercise:

Problem: $9\left(8x-3\right)-\left(-2\right)$

Solution:

$72x-25$

Exercise:

Problem: $4\left(6x-1\right)-\left(-8\right)$

Exercise:

Problem: $14\left(c-1\right)-8\left(c-6\right)$

Solution:

$6c+34$

Exercise:

Problem: $11\left(n-7\right)-5\left(n-1\right)$

Exercise:

Problem: $6\left(7y+8\right)-\left(30y-15\right)$

Solution:

$12y+63$

Exercise:

Problem: $7\left(3n+9\right)-\left(4n-13\right)$

Everyday Math

Exercise:

Problem:

Insurance copayment Carrie had to have 5 fillings done. Each filling cost \$80. Her dental insurance required her to pay 20% of the cost as a copay. Calculate Carrie's copay:

ⓐ First, by multiplying 0.20 by 80 to find her copay for each filling and then multiplying your answer by 5 to find her total copay for 5 fillings.
ⓑ Next, by multiplying [5(0.20)](80)
ⓒ Which of the properties of real numbers says that your answers to parts (a), where you multiplied 5[(0.20)(80)] and (b), where you multiplied [5(0.20)](80), should be equal?

Solution:

ⓐ \$80 ⓑ \$80 ⓒ answers will vary

Exercise:

Problem:

Cooking time Helen bought a 24-pound turkey for her family's Thanksgiving dinner and wants to know what time to put the turkey in to the oven. She wants to allow 20 minutes per pound cooking time. Calculate the length of time needed to roast the turkey:

ⓐ First, by multiplying $24\cdot 20$ to find the total number of minutes and then multiplying the answer by $\frac{1}{60}$ to convert minutes into hours.
ⓑ Next, by multiplying $24\left(20\cdot \frac{1}{60}\right)$.
ⓒ Which of the properties of real numbers says that your answers to parts (a), where you multiplied $\left(24\cdot 20\right)\frac{1}{60}$, and (b), where you multiplied $24\left(20\cdot \frac{1}{60}\right)$, should be equal?

Exercise:

Problem:

Buying by the case Trader Joe's grocery stores sold a bottle of wine they called "Two Buck Chuck" for \$1.99. They sold a case of 12 bottles for \$23.88. To find the cost of 12 bottles at \$1.99, notice that 1.99 is $2-0.01$.

ⓐ Multiply 12(1.99) by using the distributive property to multiply $12\left(2-0.01\right)$.
ⓑ Was it a bargain to buy “Two Buck Chuck” by the case?

Solution:

ⓐ \$23.88 ⓑ no, the price is the same

Exercise:

Problem:

Multi-pack purchase Adele’s shampoo sells for \$3.99 per bottle at the grocery store. At the warehouse store, the same shampoo is sold as a 3 pack for \$10.49. To find the cost of 3 bottles at \$3.99, notice that 3.99 is $4-0.01$.

ⓐ Multiply 3(3.99) by using the distributive property to multiply $3\left(4-0.01\right)$.
ⓑ How much would Adele save by buying 3 bottles at the warehouse store instead of at the grocery store?

Writing Exercises

Exercise:

Problem: In your own words, state the commutative property of addition.

Solution:

Answers may vary

Exercise:

Problem:

What is the difference between the additive inverse and the multiplicative inverse of a number?

Exercise:

Problem: Simplify $8\left(x-\frac{1}{4}\right)$ using the distributive property and explain each step.

Solution:

Answers may vary

Exercise:

Problem:

Explain how you can multiply 4(\$5.97) without paper or calculator by thinking of \$5.97 as $6-0.03$ and then using the distributive property.

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can…	Confidently	With some help	No-I don't get it!
use the commutative and associative properties.			
use the identity and inverse properties of addition and multiplication.			
use the properties of zero.			
simplify expressions using the distributive property.			

ⓑ After reviewing this checklist, what will you do to become confident for all objectives?

Glossary

additive identity
: The additive identity is the number 0; adding 0 to any number does not change its value.

additive inverse
: The opposite of a number is its additive inverse. A number and it additive inverse add to 0.

multiplicative identity
: The multiplicative identity is the number 1; multiplying 1 by any number does not change the value of the number.

multiplicative inverse
: The reciprocal of a number is its multiplicative inverse. A number and its multiplicative inverse multiply to one.

Systems of Measurement
By the end of this section, you will be able to:

- Make unit conversions in the US system
- Use mixed units of measurement in the US system
- Make unit conversions in the metric system
- Use mixed units of measurement in the metric system
- Convert between the US and the metric systems of measurement
- Convert between Fahrenheit and Celsius temperatures

Note:
A more thorough introduction to the topics covered in this section can be found in the *Prealgebra* chapter, **The Properties of Real Numbers**.

Make Unit Conversions in the U.S. System

There are two systems of measurement commonly used around the world. Most countries use the metric system. The U.S. uses a different system of measurement, usually called the **U.S. system**. We will look at the U.S. system first.

The U.S. system of measurement uses units of inch, foot, yard, and mile to measure length and pound and ton to measure weight. For capacity, the units used are cup, pint, quart, and gallons. Both the U.S. system and the metric system measure time in seconds, minutes, and hours.

The equivalencies of measurements are shown in [link]. The table also shows, in parentheses, the common abbreviations for each measurement.

U.S. System of Measurement

Length	1 foot (ft.)	=	12 inches (in.)
	1 yard (yd.)	=	3 feet (ft.)
	1 mile (mi.)	=	5,280 feet (ft.)
Volume	3 teaspoons (t)	=	1 tablespoo
	16 tablespoons (T)	=	1 cup (C)
	1 cup (C)	=	8 fluid oun
	1 pint (pt.)	=	2 cups (C)
	1 quart (qt.)	=	2 pints (pt.
	1 gallon (gal)	=	4 quarts (q
Weight	1 pound (lb.)	=	16 ounces (oz.)
	1 ton	=	2000 pounds (lb.)
Time	1 minute (min)	=	60 seconds (sec)
	1 hour (hr)	=	60 minutes (mi
	1 day	=	24 hours (hr)
	1 week (wk)	=	7 days
	1 year (yr)	=	365 days

In many real-life applications, we need to convert between units of measurement, such as feet and yards, minutes and seconds, quarts and gallons, etc. We will use the identity property of multiplication to do these conversions. We'll restate the identity property of multiplication here for easy reference.

Note:
Identity Property of Multiplication

For any real number a : $\quad a \cdot 1 = a \qquad 1 \cdot a = a$

1 is the **multiplicative identity**

To use the identity property of multiplication, we write 1 in a form that will help us convert the units. For example, suppose we want to change inches to feet. We know that 1 foot is equal to 12 inches, so we will write 1 as the fraction $\frac{1\text{ foot}}{12\text{ inches}}$. When we multiply by this fraction we do not change the value, but just change the units.

But $\frac{12\text{ inches}}{1\text{ foot}}$ also equals 1. How do we decide whether to multiply by $\frac{1\text{ foot}}{12\text{ inches}}$ or $\frac{12\text{ inches}}{1\text{ foot}}$? We choose the fraction that will make the units we want to convert *from* divide out. Treat the unit words like factors and "divide out" common units like we do common factors. If we want to convert 66 inches to feet, which multiplication will eliminate the inches?

$$66\text{ inches} \cdot \frac{1\text{ foot}}{12\text{ inches}} \quad \text{or} \quad \cancel{66\text{ inches} \cdot \frac{12\text{ inches}}{1\text{ foot}}}$$

The first form works since $66\ \cancel{\text{inches}} \cdot \frac{1\text{ foot}}{12\ \cancel{\text{inches}}}$.

The inches divide out and leave only feet. The second form does not have any units that will divide out and so will not help us.

Example:
How to Make Unit Conversions
Exercise:

Problem: MaryAnne is 66 inches tall. Convert her height into feet.

Solution:
Solution

Step 1. Multiply the measurement to be converted by 1; write 1 as a fraction relating the units given and the units needed.	Multiply 66 inches by 1, writing 1 as a fraction relating inches and feet. We need inches in the denominator so that the inches will divide out!	$66\text{ inches} \cdot 1$ $66\text{ inches} \cdot \frac{1\text{ foot}}{12\text{ inches}}$
Step 2. Multiply.	Think of 66 inches as $\frac{66\text{ inches}}{1}$.	$\frac{66\text{ inches} \cdot 1\text{ foot}}{12\text{ inches}}$
Step 3. Simplify the fraction.	Notice: inches divide out.	$\frac{66\ \cancel{\text{inches}} \cdot 1\text{ foot}}{12\ \cancel{\text{inches}}}$ $\frac{66\text{ feet}}{12}$
Step 4. Simplify.	Divide 66 by 12.	5.5 feet

Note:
Exercise:

Problem: Lexie is 30 inches tall. Convert her height to feet.

Solution:

2.5 feet

Note:
Exercise:

Problem: Rene bought a hose that is 18 yards long. Convert the length to feet.

Solution:

54 feet

Note:
Make Unit Conversions.

Multiply the measurement to be converted by 1; write 1 as a fraction relating the units given and the units needed.
Multiply.
Simplify the fraction.
Simplify.

When we use the identity property of multiplication to convert units, we need to make sure the units we want to change from will divide out. Usually this means we want the conversion fraction to have those units in the denominator.

Example:
Exercise:

Problem:

Ndula, an elephant at the San Diego Safari Park, weighs almost 3.2 tons. Convert her weight to pounds.

Solution:
Solution

We will convert 3.2 tons into pounds. We will use the identity property of multiplication, writing 1 as the fraction $\frac{2000 \text{ pounds}}{1 \text{ ton}}$.

	3.2 tons
Multiply the measurement to be converted, by 1.	$3.2 \text{ tons} \cdot 1$
Write 1 as a fraction relating tons and pounds.	$3.2 \text{ tons} \cdot \frac{2{,}000 \text{ pounds}}{1 \text{ ton}}$
Simplify.	$\frac{3.2 \cancel{\text{tons}} \cdot 2{,}000 \text{ pounds}}{1 \cancel{\text{ton}}}$
Multiply.	6,400 pounds
	Ndula weighs almost 6,400 pounds.

Note:
Exercise:

Problem: Arnold's SUV weighs about 4.3 tons. Convert the weight to pounds.

Solution:

8,600 pounds

Note:
Exercise:

Problem: The Carnival *Destiny* cruise ship weighs 51,000 tons. Convert the weight to pounds.

Solution:

102,000,000 pounds

Sometimes, to convert from one unit to another, we may need to use several other units in between, so we will need to multiply several fractions.

Example:
Exercise:

Problem:

Juliet is going with her family to their summer home. She will be away from her boyfriend for 9 weeks. Convert the time to minutes.

Solution:
Solution

To convert weeks into minutes we will convert weeks into days, days into hours, and then hours into minutes. To do this we will multiply by conversion factors of 1.

	9 weeks
Write 1 as $\frac{7\text{ days}}{1\text{ week}}$, $\frac{24\text{ hours}}{1\text{ day}}$, and $\frac{60\text{ minutes}}{1\text{ hour}}$.	$\frac{9\text{ wk}}{1} \cdot \frac{7\text{ days}}{1\text{ wk}} \cdot \frac{24\text{ hr}}{1\text{ day}} \cdot \frac{60\text{ min}}{1\text{ hr}}$
Divide out the common units.	$\frac{9\cancel{\text{ wk}}}{1} \cdot \frac{7\cancel{\text{ days}}}{1\cancel{\text{ wk}}} \cdot \frac{24\cancel{\text{ hr}}}{1\cancel{\text{ day}}} \cdot \frac{60\text{ min}}{1\cancel{\text{ hr}}}$
Multiply.	$\frac{9\cdot 7\cdot 24\cdot 60\text{ min}}{1\cdot 1\cdot 1\cdot 1}$
Multiply.	90,720 min

Juliet and her boyfriend will be apart for 90,720 minutes (although it may seem like an eternity!).

Note:
Exercise:

Problem: The distance between the earth and the moon is about 250,000 miles. Convert this length to yards.

Solution:

440,000,000 yards

Note:
Exercise:

Problem:

The astronauts of Expedition 28 on the International Space Station spend 15 weeks in space. Convert the time to minutes.

Solution:

151,200 minutes

Example:
Exercise:

Problem: How many ounces are in 1 gallon?

Solution:
Solution

We will convert gallons to ounces by multiplying by several conversion factors. Refer to [link].

	1 gallon
Multiply the measurement to be converted by 1.	$\frac{1\text{ gallon}}{1}\cdot\frac{4\text{ quarts}}{1\text{ gallon}}\cdot\frac{2\text{ pints}}{1\text{ quart}}\cdot\frac{2\text{ cups}}{1\text{ pint}}\cdot\frac{8\text{ ounces}}{1\text{ cup}}$
Use conversion factors to get to the right unit. Simplify.	$\frac{1\cancel{\text{ gallon}}}{1}\cdot\frac{4\cancel{\text{ quarts}}}{1\cancel{\text{ gallon}}}\cdot\frac{2\cancel{\text{ pints}}}{1\cancel{\text{ quart}}}\cdot\frac{2\cancel{\text{ cups}}}{1\cancel{\text{ pint}}}\cdot\frac{8\text{ ounces}}{1\cancel{\text{ cup}}}$
Multiply.	$\frac{1\cdot4\cdot2\cdot2\cdot8\text{ ounces}}{1\cdot1\cdot1\cdot1\cdot1}$
Simplify.	128 ounces There are 128 ounces in a gallon.

Note:
Exercise:

Problem: How many cups are in 1 gallon?

Solution:

16 cups

Note:
Exercise:

Problem: How many teaspoons are in 1 cup?

Solution:

48 teaspoons

Use Mixed Units of Measurement in the U.S. System

We often use mixed units of measurement in everyday situations. Suppose Joe is 5 feet 10 inches tall, stays at work for 7 hours and 45 minutes, and then eats a 1 pound 2 ounce steak for dinner—all these measurements have mixed units.

Performing arithmetic operations on measurements with mixed units of measures requires care. Be sure to add or subtract like units!

Example:
Exercise:

Problem:

Seymour bought three steaks for a barbecue. Their weights were 14 ounces, 1 pound 2 ounces and 1 pound 6 ounces. How many total pounds of steak did he buy?

Solution:
Solution

We will add the weights of the steaks to find the total weight of the steaks.

Add the ounces. Then add the pounds.	$\begin{array}{rr} & 14 \text{ ounces} \\ 1 \text{ pound} & 2 \text{ ounces} \\ + 1 \text{ pound} & 6 \text{ ounces} \\ \hline 2 \text{ pounds} & 22 \text{ ounces} \end{array}$
Convert 22 ounces to pounds and ounces.	2 pounds + 1 pound, 6 ounces
Add the pounds.	3 pounds, 6 ounces
	Seymour bought 3 pounds 6 ounces of steak.

Note:
Exercise:

Problem:

Laura gave birth to triplets weighing 3 pounds 3 ounces, 3 pounds 3 ounces, and 2 pounds 9 ounces. What was the total birth weight of the three babies?

Solution:

9 lbs. 8 oz

Note:
Exercise:

Problem:

Stan cut two pieces of crown molding for his family room that were 8 feet 7 inches and 12 feet 11 inches. What was the total length of the molding?

Solution:

21 ft. 6 in.

Example:
Exercise:

Problem:

Anthony bought four planks of wood that were each 6 feet 4 inches long. What is the total length of the wood he purchased?

Solution:
Solution

We will multiply the length of one plank to find the total length.

Multiply the inches and then the feet.	$\begin{array}{rr} \text{6 feet} & \text{4 inches} \\ \times & 4 \\ \hline \text{24 feet} & \text{16 inches} \end{array}$
Convert the 16 inches to feet. Add the feet.	$\begin{array}{l} \text{24 feet + 1 foot 4 inches} \\ \hline \text{25 feet 4 inches} \end{array}$
	Anthony bought 25 feet and 4 inches of wood.

Note:
Exercise:

Problem:

Henri wants to triple his spaghetti sauce recipe that uses 1 pound 8 ounces of ground turkey. How many pounds of ground turkey will he need?

Solution:

4 lbs. 8 oz.

Note:
Exercise:

Problem:

Joellen wants to double a solution of 5 gallons 3 quarts. How many gallons of solution will she have in all?

Solution:

11 gallons 2 qt.

Make Unit Conversions in the Metric System

In the **metric system**, units are related by powers of 10. The roots words of their names reflect this relation. For example, the basic unit for measuring length is a meter. One kilometer is 1,000 meters; the prefix *kilo* means *thousand*. One centimeter is $\frac{1}{100}$ of a meter, just like one cent is $\frac{1}{100}$ of one dollar.

The equivalencies of measurements in the metric system are shown in [link]. The common abbreviations for each measurement are given in parentheses.

Metric System of Measurement		
Length	**Mass**	**Capacity**
1 kilometer (km) = 1,000 m	1 kilogram (kg) = 1,000 g	1 kiloliter (kL) = 1,000 L
1 hectometer (hm) = 100 m	1 hectogram (hg) = 100 g	1 hectoliter (hL) = 100 L
1 dekameter (dam) = 10 m	1 dekagram (dag) = 10 g	1 dekaliter (daL) = 10 L
1 meter (m) = 1 m	1 gram (g) = 1 g	1 liter (L) = 1 L
1 decimeter (dm) = 0.1 m	1 decigram (dg) = 0.1 g	1 deciliter (dL) = 0.1 L
1 centimeter (cm) = 0.01 m	1 centigram (cg) = 0.01 g	1 centiliter (cL) = 0.01 L
1 millimeter (mm) = 0.001 m	1 milligram (mg) = 0.001 g	1 milliliter (mL) = 0.001 L
1 meter = 100 centimeters	1 gram = 100 centigrams	1 liter = 100 centiliters
1 meter = 1,000 millimeters	1 gram = 1,000 milligrams	1 liter = 1,000 milliliters

To make conversions in the metric system, we will use the same technique we did in the US system. Using the identity property of multiplication, we will multiply by a conversion factor of one to get to the correct units.

Have you ever run a 5K or 10K race? The length of those races are measured in kilometers. The metric system is commonly used in the United States when talking about the length of a race.

Example:
Exercise:

Problem: Nick ran a 10K race. How many meters did he run?

Solution:
Solution

We will convert kilometers to meters using the identity property of multiplication.

	10 kilometers
Multiply the measurement to be converted by 1.	$10 \text{ kilometers} \cdot 1$
Write 1 as a fraction relating kilometers and meters.	$10 \text{ kilometers} \cdot \frac{1{,}000 \text{ meters}}{1 \text{ kilometers}}$
Simplify.	$\frac{10 \cancel{\text{kilometers}} \cdot 1{,}000 \text{ m}}{1 \cancel{\text{kilometers}}}$
Multiply.	10,000 meters
	Nick ran 10,000 meters.

Note:
Exercise:

Problem: Sandy completed her first 5K race! How many meters did she run?

Solution:

5,000 meters

Note:
Exercise:

Problem: Herman bought a rug 2.5 meters in length. How many centimeters is the length?

Solution:

250 centimeters

Example:
Exercise:

Problem: Eleanor's newborn baby weighed 3,200 grams. How many kilograms did the baby weigh?

Solution:
Solution

We will convert grams into kilograms.

	$3{,}200 \text{ grams}$
Multiply the measurement to be converted by 1.	$3{,}200 \text{ grams} \cdot 1$
Write 1 as a function relating kilograms and grams.	$3{,}200 \text{ grams} \cdot \frac{1 \text{ kg}}{1{,}000 \text{ grams}}$
Simplify.	$3{,}200 \cancel{\text{grams}} \cdot \frac{1 \text{ kg}}{1{,}000 \cancel{\text{grams}}}$
Multiply.	$\frac{3{,}200 \text{ kilograms}}{1{,}000}$
Divide.	3.2 kilograms The baby weighed 3.2 kilograms.

Note:
Exercise:

Problem: Kari's newborn baby weighed 2,800 grams. How many kilograms did the baby weigh?

Solution:

2.8 kilograms

Note:
Exercise:

Problem:

Anderson received a package that was marked 4,500 grams. How many kilograms did this package weigh?

Solution:

4.5 kilograms

As you become familiar with the metric system you may see a pattern. Since the system is based on multiples of ten, the calculations involve multiplying by multiples of ten. We have learned how to simplify these calculations by just moving the decimal.

To multiply by 10, 100, or 1,000, we move the decimal to the right one, two, or three places, respectively. To multiply by 0.1, 0.01, or 0.001, we move the decimal to the left one, two, or three places, respectively.

We can apply this pattern when we make measurement conversions in the metric system. In [link], we changed 3,200 grams to kilograms by multiplying by $\frac{1}{1000}$ (or 0.001). This is the same as moving the decimal three places to the left.

$3{,}200 \cdot \frac{1}{1{,}000}$	3,200.
3.2	3.2

Example:
Exercise:

Problem: Convert ⓐ 350 L to kiloliters ⓑ 4.1 L to milliliters.

Solution:
Solution

ⓐ We will convert liters to kiloliters. In [link], we see that 1 kiloliter $=$ 1,000 liters.

	350 L
Multiply by 1, writing 1 as a fraction relating liters to kiloliters.	$350\text{ L} \cdot \frac{1\text{ kL}}{1{,}000\text{ L}}$
Simplify.	$350\cancel{\text{ L}} \cdot \frac{1\text{ kL}}{1{,}000\cancel{\text{ L}}}$
Move the decimal 3 units to the left. (350.)	0.35 kL

ⓑ We will convert liters to milliliters. From [link] we see that 1 liter $=$ 1,000 milliliters.

	4.1 L
Multiply by 1, writing 1 as a fraction relating liters to milliliters.	$4.1\text{ L}\cdot\frac{1{,}000\text{ mL}}{1\text{ L}}$
Simplify.	$4.1\,\cancel{\text{L}}\cdot\frac{1{,}000\text{ mL}}{1\,\cancel{\text{L}}}$
Move the decimal 3 units to the right.	4.100 mL
	4,100 mL

Note:
Exercise:

Problem: Convert: ⓐ 725 L to kiloliters ⓑ 6.3 L to milliliters

Solution:

ⓐ 7,250 kiloliters ⓑ 6,300 milliliters

Note:
Exercise:

Problem: Convert: ⓐ 350 hL to liters ⓑ 4.1 L to centiliters

Solution:

ⓐ 35,000 liters ⓑ 410 centiliters

Use Mixed Units of Measurement in the Metric System

Performing arithmetic operations on measurements with mixed units of measures in the metric system requires the same care we used in the US system. But it may be easier because of the relation of the units to the powers of 10. Make sure to add or subtract like units.

Example:

Exercise:

Problem:

Ryland is 1.6 meters tall. His younger brother is 85 centimeters tall. How much taller is Ryland than his younger brother?

Solution:
Solution

We can convert both measurements to either centimeters or meters. Since meters is the larger unit, we will subtract the lengths in meters. We convert 85 centimeters to meters by moving the decimal 2 places to the left.
Equation:

$$\begin{array}{lr} \text{Write the 85 centimeters as meters.} & 1.60\text{ m} \\ & \underline{-0.85\text{ m}} \\ & 0.75\text{ m} \end{array}$$

Ryland is 0.75 m taller than his brother.

Note:
Exercise:

Problem:

Mariella is 1.58 meters tall. Her daughter is 75 centimeters tall. How much taller is Mariella than her daughter? Write the answer in centimeters.

Solution:

83 centimeters

Note:
Exercise:

Problem:

The fence around Hank’s yard is 2 meters high. Hank is 96 centimeters tall. How much shorter than the fence is Hank? Write the answer in meters.

Solution:

1.04 meters

Example:
Exercise:

Problem:

Dena’s recipe for lentil soup calls for 150 milliliters of olive oil. Dena wants to triple the recipe. How many liters of olive oil will she need?

Solution:
Solution

We will find the amount of olive oil in millileters then convert to liters.

	Triple 150 mL
Translate to algebra.	$3 \cdot 150\,\text{mL}$
Multiply.	450 mL
Convert to liters.	$450 \cdot \frac{0.001\text{ L}}{1\text{ mL}}$
Simplify.	0.45 L

Dena needs 0.45 liters of olive oil.

Note:
Exercise:

Problem:

A recipe for Alfredo sauce calls for 250 milliliters of milk. Renata is making pasta with Alfredo sauce for a big party and needs to multiply the recipe amounts by 8. How many liters of milk will she need?

Solution:

2 liters

Note:
Exercise:

Problem:

To make one pan of baklava, Dorothea needs 400 grams of filo pastry. If Dorothea plans to make 6 pans of baklava, how many kilograms of filo pastry will she need?

Solution:

2.4 kilograms

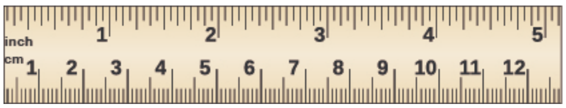
Convert Between the U.S. and the Metric Systems of Measurement

Many measurements in the United States are made in metric units. Our soda may come in 2-liter bottles, our calcium may come in 500-mg capsules, and we may run a 5K race. To work easily in both systems, we need to be able to convert between the two systems.

[link] shows some of the most common conversions.

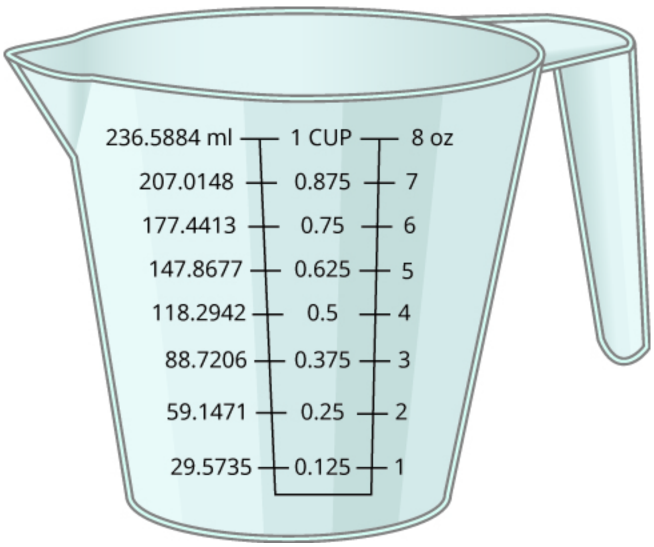
Conversion Factors Between U.S. and Metric Systems		
Length	**Mass**	**Capacity**
1 in. = 2.54 cm 1 ft. = 0.305 m 1 yd. = 0.914 m 1 mi. = 1.61 km 1 m = 3.28 ft.	1 lb. = 0.45 kg 1 oz. = 28 g 1 kg = 2.2 lb.	1 qt. = 0.95 L 1 fl. oz. = 30 mL 1 L = 1.06 qt.

[link] shows how inches and centimeters are related on a ruler.



This ruler shows inches and centimeters.

[link] shows the ounce and milliliter markings on a measuring cup.



This measuring cup shows ounces and milliliters.

[link] shows how pounds and kilograms marked on a bathroom scale.

This scale shows pounds and kilograms.

We make conversions between the systems just as we do within the systems—by multiplying by unit conversion factors.

Example:
Exercise:

Problem:

Lee’s water bottle holds 500 mL of water. How many ounces are in the bottle? Round to the nearest tenth of an ounce.

Solution:
Solution

	500 mL
Multiply by a unit conversion factor relating mL and ounces.	$500\text{ milliliters}\cdot\frac{1\text{ ounce}}{30\text{ milliliters}}$
Simplify.	$\frac{50\text{ ounce}}{30}$
Divide.	16.7 ounces.
	The water bottle has 16.7 ounces.

Note:
Exercise:

Problem: How many quarts of soda are in a 2-L bottle?

Solution:

2.12 quarts

Note:
Exercise:

Problem: How many liters are in 4 quarts of milk?

Solution:

3.8 liters

Example:
Exercise:

Problem:

Soleil was on a road trip and saw a sign that said the next rest stop was in 100 kilometers. How many miles until the next rest stop?

Solution:
Solution

	100 kilometers
Multiply by a unit conversion factor relating km and mi.	$100 \text{ kilometers} \cdot \frac{1 \text{ mile}}{1.61 \text{ kilometer}}$
Simplify.	$\frac{100 \text{ miles}}{1.61}$
Divide.	62 miles
	Soleil will travel 62 miles.

Note:
Exercise:

Problem: The height of Mount Kilimanjaro is 5,895 meters. Convert the height to feet.

Solution:

19,335.6 feet

Note:
Exercise:

Problem:

The flight distance from New York City to London is 5,586 kilometers. Convert the distance to miles.

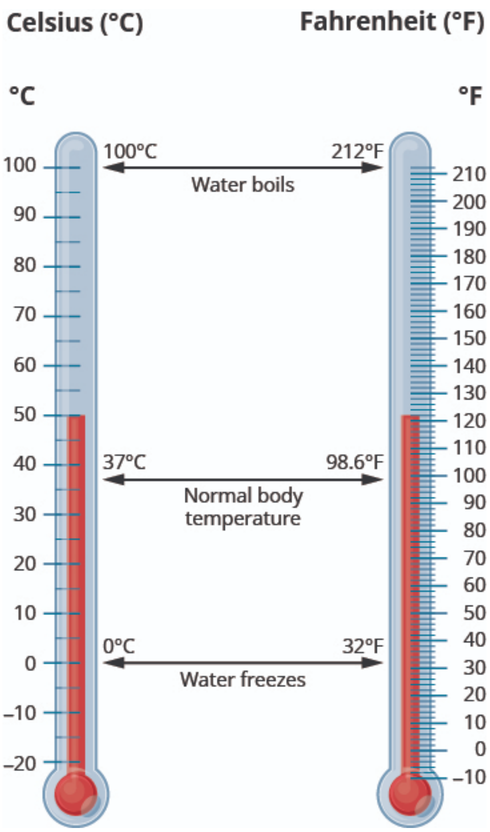
Solution:

8,993.46 km

Convert between Fahrenheit and Celsius Temperatures

Have you ever been in a foreign country and heard the weather forecast? If the forecast is for 22°C, what does that mean?

The U.S. and metric systems use different scales to measure temperature. The U.S. system uses degrees Fahrenheit, written $^\circ\text{F}$. The metric system uses degrees Celsius, written $^\circ\text{C}$. [link] shows the relationship between the two systems.



The diagram shows normal body temperature, along with the freezing and boiling temperatures of water in degrees Fahrenheit and degrees Celsius.

Note:
Temperature Conversion
To convert from Fahrenheit temperature, F, to Celsius temperature, C, use the formula
Equation:

$$C = \frac{5}{9}(F - 32).$$

To convert from Celsius temperature, C, to Fahrenheit temperature, F, use the formula
Equation:

$$F = \frac{9}{5}C + 32.$$

Example:
Exercise:

Problem: Convert 50° Fahrenheit into degrees Celsius.

Solution:
Solution

We will substitute 50°F into the formula to find C.

	$C = \frac{5}{9}(F - 32)$
Substitute 50 for F.	$C = \frac{5}{9}(50 - 32)$
Simplify in parentheses.	$C = \frac{5}{9}(18)$
Multiply.	$C = 10$
	So we found that 50°F is equivalent to 10°C.

Note:
Exercise:

Problem: Convert the Fahrenheit temperature to degrees Celsius: 59° Fahrenheit.

Solution:

15°C

Note:
Exercise:

Problem: Convert the Fahrenheit temperature to degrees Celsius: 41° Fahrenheit.

Solution:

$5^\circ\mathrm{C}$

Example:
Exercise:

Problem:

While visiting Paris, Woody saw the temperature was 20° Celsius. Convert the temperature into degrees Fahrenheit.

Solution:
Solution

We will substitute $20^\circ\mathrm{C}$ into the formula to find F.

	$F = \frac{9}{5}C + 32$
Substitute 20 for C.	$F = \frac{9}{5}(20) + 32$
Multiply.	$F = 36 + 32$
Add.	$F = 68$
	So we found that 20°C is equivalent to 68°F.

Note:
Exercise:

Problem:

Convert the Celsius temperature to degrees Fahrenheit: the temperature in Helsinki, Finland, was 15° Celsius.

Solution:

$59^{\circ}F$

Note:
Exercise:

Problem:

Convert the Celsius temperature to degrees Fahrenheit: the temperature in Sydney, Australia, was 10° Celsius.

Solution:

$50^{\circ}F$

Key Concepts

- **Metric System of Measurement**
  - **Length**

$$\begin{array}{lcl} 1 \text{ kilometer (km)} & = & 1{,}000 \text{ m} \\ 1 \text{ hectometer (hm)} & = & 100 \text{ m} \\ 1 \text{ dekameter (dam)} & = & 10 \text{ m} \\ 1 \text{ meter (m)} & = & 1 \text{ m} \\ 1 \text{ decimeter (dm)} & = & 0.1 \text{ m} \\ 1 \text{ centimeter (cm)} & = & 0.01 \text{ m} \\ 1 \text{ millimeter (mm)} & = & 0.001 \text{ m} \\ 1 \text{ meter} & = & 100 \text{ centimeters} \\ 1 \text{ meter} & = & 1{,}000 \text{ millimeters} \end{array}$$

  - **Mass**

$$\begin{array}{lcl} 1 \text{ kilogram (kg)} & = & 1{,}000 \text{ g} \\ 1 \text{ hectogram (hg)} & = & 100 \text{ g} \\ 1 \text{ dekagram (dag)} & = & 10 \text{ g} \\ 1 \text{ gram (g)} & = & 1 \text{ g} \\ 1 \text{ decigram (dg)} & = & 0.1 \text{ g} \\ 1 \text{ centigram (cg)} & = & 0.01 \text{ g} \\ 1 \text{ milligram (mg)} & = & 0.001 \text{ g} \\ 1 \text{ gram} & = & 100 \text{ centigrams} \\ 1 \text{ gram} & = & 1{,}000 \text{ milligrams} \end{array}$$

- **Capacity**

1 kiloliter (kL)	=	1,000 L
1 hectoliter (hL)	=	100 L
1 dekaliter (daL)	=	10 L
1 liter (L)	=	1 L
1 deciliter (dL)	=	0.1 L
1 centiliter (cL)	=	0.01 L
1 milliliter (mL)	=	0.001 L
1 liter	=	100 centiliters
1 liter	=	1,000 milliliters

- **Temperature Conversion**
  - To convert from Fahrenheit temperature, F, to Celsius temperature, C, use the formula $C = \frac{5}{9}(F - 32)$
  - To convert from Celsius temperature, C, to Fahrenheit temperature, F, use the formula $F = \frac{9}{5}C + 32$

Section Exercises

Practice Makes Perfect

Make Unit Conversions in the U.S. System

In the following exercises, convert the units.

Exercise:

Problem: A park bench is 6 feet long. Convert the length to inches.

Solution:

72 inches

Exercise:

Problem: A floor tile is 2 feet wide. Convert the width to inches.

Exercise:

Problem: A ribbon is 18 inches long. Convert the length to feet.

Solution:

1.5 feet

Exercise:

Problem: Carson is 45 inches tall. Convert his height to feet.

Exercise:

Problem: A football field is 160 feet wide. Convert the width to yards.

Solution:

$53\frac{1}{3}$ yards

Exercise:

Problem:

On a baseball diamond, the distance from home plate to first base is 30 yards. Convert the distance to feet.

Exercise:

Problem: Ulises lives 1.5 miles from school. Convert the distance to feet.

Solution:

7,920 feet

Exercise:

Problem: Denver, Colorado, is 5,183 feet above sea level. Convert the height to miles.

Exercise:

Problem: A killer whale weighs 4.6 tons. Convert the weight to pounds.

Solution:

9,200 pounds

Exercise:

Problem: Blue whales can weigh as much as 150 tons. Convert the weight to pounds.

Exercise:

Problem: An empty bus weighs 35,000 pounds. Convert the weight to tons.

Solution:

$17\frac{1}{2}$ tons

Exercise:

Problem: At take-off, an airplane weighs 220,000 pounds. Convert the weight to tons.

Exercise:

Problem: Rocco waited $1\frac{1}{2}$ hours for his appointment. Convert the time to seconds.

Solution:

5,400 s

Exercise:

Problem: Misty's surgery lasted $2\frac{1}{4}$ hours. Convert the time to seconds.

Exercise:

Problem: How many teaspoons are in a pint?

Solution:

96 teaspoons

Exercise:

Problem: How many tablespoons are in a gallon?

Exercise:

Problem: JJ’s cat, Posy, weighs 14 pounds. Convert her weight to ounces.

Solution:

224 ounces

Exercise:

Problem: April’s dog, Beans, weighs 8 pounds. Convert his weight to ounces.

Exercise:

Problem: Crista will serve 20 cups of juice at her son’s party. Convert the volume to gallons.

Solution:

$1\frac{1}{4}$ gallons

Exercise:

Problem: Lance needs 50 cups of water for the runners in a race. Convert the volume to gallons.

Exercise:

Problem: Jon is 6 feet 4 inches tall. Convert his height to inches.

Solution:

26 in.

Exercise:

Problem: Faye is 4 feet 10 inches tall. Convert her height to inches.

Exercise:

Problem: The voyage of the *Mayflower* took 2 months and 5 days. Convert the time to days.

Solution:

65 days

Exercise:

Problem: Lynn’s cruise lasted 6 days and 18 hours. Convert the time to hours.

Exercise:

Problem: Baby Preston weighed 7 pounds 3 ounces at birth. Convert his weight to ounces.

Solution:

115 ounces

Exercise:

Problem: Baby Audrey weighted 6 pounds 15 ounces at birth. Convert her weight to ounces.

Use Mixed Units of Measurement in the U.S. System

In the following exercises, solve.

Exercise:

Problem:

Eli caught three fish. The weights of the fish were 2 pounds 4 ounces, 1 pound 11 ounces, and 4 pounds 14 ounces. What was the total weight of the three fish?

Solution:

8 lbs. 13 oz.

Exercise:

Problem:

Judy bought 1 pound 6 ounces of almonds, 2 pounds 3 ounces of walnuts, and 8 ounces of cashews. How many pounds of nuts did Judy buy?

Exercise:

Problem:

One day Anya kept track of the number of minutes she spent driving. She recorded 45, 10, 8, 65, 20, and 35. How many hours did Anya spend driving?

Solution:

3.05 hours

Exercise:

Problem:

Last year Eric went on 6 business trips. The number of days of each was 5, 2, 8, 12, 6, and 3. How many weeks did Eric spend on business trips last year?

Exercise:

Problem:

Renee attached a 6 feet 6 inch extension cord to her computer's 3 feet 8 inch power cord. What was the total length of the cords?

Solution:

10 ft. 2 in.

Exercise:

Problem:

Fawzi's SUV is 6 feet 4 inches tall. If he puts a 2 feet 10 inch box on top of his SUV, what is the total height of the SUV and the box?

Exercise:

Problem:

Leilani wants to make 8 placemats. For each placemat she needs 18 inches of fabric. How many yards of fabric will she need for the 8 placemats?

Solution:

4 yards

Exercise:

Problem:

Mireille needs to cut 24 inches of ribbon for each of the 12 girls in her dance class. How many yards of ribbon will she need altogether?

Make Unit Conversions in the Metric System

In the following exercises, convert the units.

Exercise:

Problem: Ghalib ran 5 kilometers. Convert the length to meters.

Solution:

5,000 meters

Exercise:

Problem: Kitaka hiked 8 kilometers. Convert the length to meters.

Exercise:

Problem: Estrella is 1.55 meters tall. Convert her height to centimeters.

Solution:

155 centimeters

Exercise:

Problem: The width of the wading pool is 2.45 meters. Convert the width to centimeters.

Exercise:

Problem: Mount Whitney is 3,072 meters tall. Convert the height to kilometers.

Solution:

3.072 kilometers

Exercise:

Problem: The depth of the Mariana Trench is 10,911 meters. Convert the depth to kilometers.

Exercise:

Problem: June’s multivitamin contains 1,500 milligrams of calcium. Convert this to grams.

Solution:

1.5 grams

Exercise:

Problem: A typical ruby-throated hummingbird weights 3 grams. Convert this to milligrams.

Exercise:

Problem: One stick of butter contains 91.6 grams of fat. Convert this to milligrams.

Solution:

91,600 milligrams

Exercise:

Problem: One serving of gourmet ice cream has 25 grams of fat. Convert this to milligrams.

Exercise:

Problem: The maximum mass of an airmail letter is 2 kilograms. Convert this to grams.

Solution:

2,000 grams

Exercise:

Problem: Dimitri's daughter weighed 3.8 kilograms at birth. Convert this to grams.

Exercise:

Problem: A bottle of wine contained 750 milliliters. Convert this to liters.

Solution:

0.75 liters

Exercise:

Problem: A bottle of medicine contained 300 milliliters. Convert this to liters.

Use Mixed Units of Measurement in the Metric System

In the following exercises, solve.

Exercise:

Problem: Matthias is 1.8 meters tall. His son is 89 centimeters tall. How much taller is Matthias than his son?

Solution:

91 centimeters

Exercise:

Problem:

Stavros is 1.6 meters tall. His sister is 95 centimeters tall. How much taller is Stavros than his sister?

Exercise:

Problem:

A typical dove weighs 345 grams. A typical duck weighs 1.2 kilograms. What is the difference, in grams, of the weights of a duck and a dove?

Solution:

855 grams

Exercise:

Problem:

Concetta had a 2-kilogram bag of flour. She used 180 grams of flour to make biscotti. How many kilograms of flour are left in the bag?

Exercise:

Problem:

Harry mailed 5 packages that weighed 420 grams each. What was the total weight of the packages in kilograms?

Solution:

2.1 kilograms

Exercise:

Problem:

One glass of orange juice provides 560 milligrams of potassium. Linda drinks one glass of orange juice every morning. How many grams of potassium does Linda get from her orange juice in 30 days?

Exercise:

Problem:

Jonas drinks 200 milliliters of water 8 times a day. How many liters of water does Jonas drink in a day?

Solution:

1.6 liters

Exercise:

Problem:

One serving of whole grain sandwich bread provides 6 grams of protein. How many milligrams of protein are provided by 7 servings of whole grain sandwich bread?

Convert Between the U.S. and the Metric Systems of Measurement

In the following exercises, make the unit conversions. Round to the nearest tenth.

Exercise:

Problem: Bill is 75 inches tall. Convert his height to centimeters.

Solution:

190.5 centimeters

Exercise:

Problem: Frankie is 42 inches tall. Convert his height to centimeters.

Exercise:

Problem: Marcus passed a football 24 yards. Convert the pass length to meters

Solution:

21.9 meters

Exercise:

Problem: Connie bought 9 yards of fabric to make drapes. Convert the fabric length to meters.

Exercise:

Problem:

Each American throws out an average of 1,650 pounds of garbage per year. Convert this weight to kilograms.

Solution:

742.5 kilograms

Exercise:

Problem:

An average American will throw away 90,000 pounds of trash over his or her lifetime. Convert this weight to kilograms.

Exercise:

Problem: A 5K run is 5 kilometers long. Convert this length to miles.

Solution:

3.1 miles

Exercise:

Problem: Kathryn is 1.6 meters tall. Convert her height to feet.

Exercise:

Problem: Dawn's suitcase weighed 20 kilograms. Convert the weight to pounds.

Solution:

44 pounds

Exercise:

Problem: Jackson's backpack weighed 15 kilograms. Convert the weight to pounds.

Exercise:

Problem: Ozzie put 14 gallons of gas in his truck. Convert the volume to liters.

Solution:

53.2 liters

Exercise:

Problem: Bernard bought 8 gallons of paint. Convert the volume to liters.

Convert between Fahrenheit and Celsius Temperatures

In the following exercises, convert the Fahrenheit temperatures to degrees Celsius. Round to the nearest tenth.

Exercise:

Problem: 86° Fahrenheit

Solution:

$30^\circ C$

Exercise:

Problem: 77° Fahrenheit

Exercise:

Problem: 104° Fahrenheit

Solution:

$40^\circ C$

Exercise:

Problem: 14° Fahrenheit

Exercise:

Problem: 72° Fahrenheit

Solution:

$22.2^\circ C$

Exercise:

Problem: 4° Fahrenheit

Exercise:

Problem: 0° Fahrenheit

Solution:

$-17.8^\circ C$

Exercise:

Problem: 120° Fahrenheit

In the following exercises, convert the Celsius temperatures to degrees Fahrenheit. Round to the nearest tenth.

Exercise:

Problem: 5° Celsius

Solution:

41°F

Exercise:

Problem: 25° Celsius

Exercise:

Problem: -10° Celsius

Solution:

14°F

Exercise:

Problem: -15° Celsius

Exercise:

Problem: 22° Celsius

Solution:

71.6°F

Exercise:

Problem: 8° Celsius

Exercise:

Problem: 43° Celsius

Solution:

109.4°F

Exercise:

Problem: 16° Celsius

Everyday Math

Exercise:

Problem:

Nutrition Julian drinks one can of soda every day. Each can of soda contains 40 grams of sugar. How many kilograms of sugar does Julian get from soda in 1 year?

Solution:

14.6 kilograms

Exercise:

Problem:

Reflectors The reflectors in each lane-marking stripe on a highway are spaced 16 yards apart. How many reflectors are needed for a one mile long lane-marking stripe?

Writing Exercises

Exercise:

Problem: Some people think that 65° to 75° Fahrenheit is the ideal temperature range.

ⓐ What is your ideal temperature range? Why do you think so?
ⓑ Convert your ideal temperatures from Fahrenheit to Celsius.

Solution:

Answers may vary.

Exercise:

Problem:

ⓐ Did you grow up using the U.S. or the metric system of measurement?
ⓑ Describe two examples in your life when you had to convert between the two systems of measurement.

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
define U.S. units of measurement and convert from one unit to another.			
use U.S units of measurement.			
define metric units of measurement and convert from one unit to another.			
use metric units of measurement.			
convert between the U.S. and the metric system of measurement.			
convert between Fahrenheit and Celsius temperatures.			

ⓑ Overall, after looking at the checklist, do you think you are well-prepared for the next Chapter? Why or why not?

Chapter Review Exercises

Introduction to Whole Numbers

Use Place Value with Whole Number

In the following exercises find the place value of each digit.
Exercise:

Problem: 26,915

ⓐ 1
ⓑ 2
ⓒ 9
ⓓ 5
ⓔ 6

Solution:

ⓐ tens ⓑ ten thousands ⓒ hundreds ⓓ ones ⓔ thousands

Exercise:

Problem: 359,417

ⓐ 9
ⓑ 3
ⓒ 4
ⓓ 7
ⓔ 1

Exercise:

Problem: 58,129,304

ⓐ 5
ⓑ 0
ⓒ 1

ⓓ 8
ⓔ 2

Solution:

ⓐ ten millions ⓑ tens ⓒ hundred thousands ⓓ millions ⓔ ten thousands

Exercise:

Problem: 9,430,286,157

ⓐ 6
ⓑ 4
ⓒ 9
ⓓ 0
ⓔ 5

In the following exercises, name each number.

Exercise:

Problem: 6,104

Solution:

six thousand, one hundred four

Exercise:

Problem: 493,068

Exercise:

Problem: 3,975,284

Solution:

three million, nine hundred seventy-five thousand, two hundred eighty-four

Exercise:

Problem: 85,620,435

In the following exercises, write each number as a whole number using digits.

Exercise:

Problem: three hundred fifteen

Solution:

315

Exercise:

Problem: sixty-five thousand, nine hundred twelve

Exercise:

Problem: ninety million, four hundred twenty-five thousand, sixteen

Solution:

90,425,016

Exercise:

Problem: one billion, forty-three million, nine hundred twenty-two thousand, three hundred eleven

In the following exercises, round to the indicated place value.

Exercise:

Problem: Round to the nearest ten.

ⓐ 407 ⓑ 8,564

Solution:

ⓐ 410 ⓑ 8,560

Exercise:

Problem: Round to the nearest hundred.

ⓐ 25,846 ⓑ 25,864

In the following exercises, round each number to the nearest ⓐ hundred ⓑ thousand ⓒ ten thousand.

Exercise:

Problem: 864,951

Solution:

ⓐ 865,000 ⓑ 865,000 ⓒ 860,000

Exercise:

Problem: 3,972,849

Identify Multiples and Factors

In the following exercises, use the divisibility tests to determine whether each number is divisible by 2, by 3, by 5, by 6, and by 10.

Exercise:

Problem: 168

Solution:

by $2, 3, 6$

Exercise:

Problem: 264

Exercise:

Problem: 375

Solution:

by $3, 5$

Exercise:

Problem: 750

Exercise:

Problem: 1430

Solution:

by $2, 5, 10$

Exercise:

Problem: 1080

Find Prime Factorizations and Least Common Multiples

In the following exercises, find the prime factorization.

Exercise:

Problem: 420

Solution:

$2 \cdot 2 \cdot 3 \cdot 5 \cdot 7$

Exercise:

Problem: 115

Exercise:

Problem: 225

Solution:

$3 \cdot 3 \cdot 5 \cdot 5$

Exercise:

Problem: 2475

Exercise:

Problem: 1560

Solution:

$2 \cdot 2 \cdot 2 \cdot 3 \cdot 5 \cdot 13$

Exercise:

Problem: 56

Exercise:

Problem: 72

Solution:

$2 \cdot 2 \cdot 2 \cdot 3 \cdot 3$

Exercise:

Problem: 168

Exercise:

Problem: 252

Solution:

$2 \cdot 2 \cdot 3 \cdot 3 \cdot 7$

Exercise:

Problem: 391

In the following exercises, find the least common multiple of the following numbers using the multiples method.

Exercise:

Problem: 6,15

Solution:

30

Exercise:

Problem: 60, 75

In the following exercises, find the least common multiple of the following numbers using the prime factors method.

Exercise:

Problem: 24, 30

Solution:

120

Exercise:

Problem: 70, 84

Use the Language of Algebra

Use Variables and Algebraic Symbols

In the following exercises, translate the following from algebra to English.
Exercise:

Problem: $25 - 7$

Solution:

25 minus 7, the difference of twenty-five and seven

Exercise:

Problem: $5 \cdot 6$

Exercise:

Problem: $45 \div 5$

Solution:

45 divided by 5, the quotient of forty-five and five

Exercise:

Problem: $x + 8$

Exercise:

Problem: $42 \geq 27$

Solution:

forty-two is greater than or equal to twenty-seven

Exercise:

Problem: $3n = 24$

Exercise:

Problem: $3 \leq 20 \div 4$

Solution:

3 is less than or equal to 20 divided by 4, three is less than or equal to the quotient of twenty and four

Exercise:

Problem: $a \neq 7 \cdot 4$

In the following exercises, determine if each is an expression or an equation.
Exercise:

Problem: $6 \cdot 3 + 5$

Solution:

expression

Exercise:

Problem: $y - 8 = 32$

Simplify Expressions Using the Order of Operations

In the following exercises, simplify each expression.

Exercise:

Problem: 3^5

Solution:

243

Exercise:

Problem: 10^8

In the following exercises, simplify

Exercise:

Problem: $6 + 10/2 + 2$

Solution:

13

Exercise:

Problem: $9 + 12/3 + 4$

Exercise:

Problem: $20 \div (4 + 6) \cdot 5$

Solution:

10

Exercise:

Problem: $33 \div (3 + 8) \cdot 2$

Exercise:

Problem: $4^2 + 5^2$

Solution:

41

Exercise:

Problem: $(4+5)^2$

Evaluate an Expression

In the following exercises, evaluate the following expressions.
Exercise:

Problem: $9x+7$ when $x=3$

Solution:

34

Exercise:

Problem: $5x-4$ when $x=6$

Exercise:

Problem: x^4 when $x=3$

Solution:

81

Exercise:

Problem: 3^x when $x=3$

Exercise:

Problem: x^2+5x-8 when $x=6$

Solution:

58

Exercise:

Problem: $2x+4y-5$ when $x=7, y=8$

Simplify Expressions by Combining Like Terms

In the following exercises, identify the coefficient of each term.
Exercise:

Problem: $12n$

Solution:

12

Exercise:

Problem: $9x^2$

In the following exercises, identify the like terms.
Exercise:

Problem: $3n, n^2, 12, 12p^2, 3, 3n^2$

Solution:

12 and 3, n^2 and $3n^2$

Exercise:

Problem: $5, 18r^2, 9s, 9r, 5r^2, 5s$

In the following exercises, identify the terms in each expression.
Exercise:

Problem: $11x^2 + 3x + 6$

Solution:

$11x^2, 3x, 6$

Exercise:

Problem: $22y^3 + y + 15$

In the following exercises, simplify the following expressions by combining like terms.
Exercise:

Problem: $17a + 9a$

Solution:

$26a$

Exercise:

Problem: $18z + 9z$

Exercise:

Problem: $9x + 3x + 8$

Solution:

$12x + 8$

Exercise:

Problem: $8a + 5a + 9$

Exercise:

Problem: $7p + 6 + 5p - 4$

Solution:

$12p + 2$

Exercise:

Problem: $8x + 7 + 4x - 5$

Translate an English Phrase to an Algebraic Expression

In the following exercises, translate the following phrases into algebraic expressions.

Exercise:

Problem: the sum of 8 and 12

Solution:

$8 + 12$

Exercise:

Problem: the sum of 9 and 1

Exercise:

Problem: the difference of x and 4

Solution:

$x - 4$

Exercise:

Problem: the difference of x and 3

Exercise:

Problem: the product of 6 and y

Solution:

$6y$

Exercise:

Problem: the product of 9 and y

Exercise:

Problem:

Adele bought a skirt and a blouse. The skirt cost $15 more than the blouse. Let b represent the cost of the blouse. Write an expression for the cost of the skirt.

Solution:

$b + 15$

Exercise:

Problem:

Marcella has 6 fewer boy cousins than girl cousins. Let g represent the number of girl cousins. Write an expression for the number of boy cousins.

Add and Subtract Integers

Use Negatives and Opposites of Integers

In the following exercises, order each of the following pairs of numbers, using < or >.

Exercise:

Problem: ⓐ 6 ___ 2
ⓑ -7 ___ 4
ⓒ -9 ___ -1
ⓓ 9 ___ -3

Solution:

ⓐ > ⓑ < ⓒ < ⓓ >

Exercise:

Problem: ⓐ -5 ___ 1
ⓑ -4 ___ -9
ⓒ 6 ___ 10
ⓓ 3 ___ -8

In the following exercises,, find the opposite of each number.

Exercise:

Problem: ⓐ -8 ⓑ 1

Solution:

ⓐ 8 ⓑ -1

Exercise:

Problem: ⓐ -2 ⓑ 6

In the following exercises, simplify.

Exercise:

Problem: $-(-19)$

Solution:

19

Exercise:

Problem: $-(-53)$

In the following exercises, simplify.
Exercise:

$-m$ when
ⓐ $m=3$
Problem: ⓑ $m=-3$

Solution:

ⓐ -3 ⓑ 3

Exercise:

$-p$ when
ⓐ $p=6$
Problem: ⓑ $p=-6$

Simplify Expressions with Absolute Value

In the following exercises,, simplify.
Exercise:

Problem: ⓐ $|7|$ ⓑ $|-25|$ ⓒ $|0|$

Solution:

ⓐ 7 ⓑ 25 ⓒ 0

Exercise:

Problem: ⓐ $|5|$ ⓑ $|0|$ ⓒ $|-19|$

In the following exercises, fill in <, >, or = for each of the following pairs of numbers.
Exercise:

ⓐ -8 ___ $|-8|$
Problem: ⓑ $-|-2|$ ___ -2

Solution:

ⓐ < ⓑ =

Exercise:

ⓐ $|-3|$ ___ $-|-3|$
Problem: ⓑ 4 ___ $-|-4|$

In the following exercises, simplify.

Exercise:

Problem: $|8-4|$

Solution:

4

Exercise:

Problem: $|9-6|$

Exercise:

Problem: $8\left(14-2\left|-2\right|\right)$

Solution:

80

Exercise:

Problem: $6\left(13-4\left|-2\right|\right)$

In the following exercises, evaluate.

Exercise:

Problem: ⓐ $|x|$ when $x=-28$ ⓑ

Solution:

ⓐ 28 ⓑ 15

Exercise:

Problem: ⓐ $|y|$ when $y=-37$
ⓑ $|-z|$ when $z=-24$

Add Integers

In the following exercises, simplify each expression.

Exercise:

Problem: $-200+65$

Solution:

-135

Exercise:

Problem: $-150+45$

Exercise:

Problem: $2+(-8)+6$

Solution:

0

Exercise:

Problem: $4+(-9)+7$

Exercise:

Problem: $140+(-75)+67$

Solution:

132

Exercise:

Problem: $-32+24+(-6)+10$

Subtract Integers

In the following exercises, simplify.

Exercise:

Problem: $9-3$

Solution:

6

Exercise:

Problem: $-5-(-1)$

Exercise:

Problem: ⓐ $15-6$ ⓑ $15+(-6)$

Solution:

ⓐ 9 ⓑ 9

Exercise:

Problem: ⓐ $12-9$ ⓑ $12+(-9)$

Exercise:

Problem: ⓐ $8-(-9)$ ⓑ $8+9$

Solution:

ⓐ 17 ⓑ 17

Exercise:

Problem: ⓐ $4-(-4)$ ⓑ $4+4$

In the following exercises, simplify each expression.

Exercise:

Problem: $10-(-19)$

Solution:

29

Exercise:

Problem: $11-(-18)$

Exercise:

Problem: $31-79$

Solution:

-48

Exercise:

Problem: $39-81$

Exercise:

Problem: $-31-11$

Solution:

-42

Exercise:

Problem: $-32-18$

Exercise:

Problem: $-15-(-28)+5$

Solution:

18

Exercise:

Problem: $71+(-10)-8$

Exercise:

Problem: $-16-(-4+1)-7$

Solution:

-20

Exercise:

Problem: $-15-(-6+4)-3$

Multiply Integers

In the following exercises, multiply.

Exercise:

Problem: $-5\,(7)$

Solution:

-35

Exercise:

Problem: $-8\,(6)$

Exercise:

Problem: $-18\,(-2)$

Solution:

36

Exercise:

Problem: $-10\,(-6)$

Divide Integers

In the following exercises, divide.

Exercise:

Problem: $-28\div 7$

Solution:

-4

Exercise:

Problem: $56\div(-7)$

Exercise:

Problem: $-120\div(-20)$

Solution:

6

Exercise:

Problem: $-200 \div 25$

Simplify Expressions with Integers

In the following exercises, simplify each expression.

Exercise:

Problem: $-8\left(-2\right)-3\left(-9\right)$

Solution:

43

Exercise:

Problem: $-7\left(-4\right)-5\left(-3\right)$

Exercise:

Problem: $\left(-5\right)^{3}$

Solution:

-125

Exercise:

Problem: $\left(-4\right)^{3}$

Exercise:

Problem: $-4\cdot 2\cdot 11$

Solution:

-88

Exercise:

Problem: $-5\cdot 3\cdot 10$

Exercise:

Problem: $-10\left(-4\right)\div\left(-8\right)$

Solution:

-5

Exercise:

Problem: $-8\left(-6\right)\div\left(-4\right)$

Exercise:

Problem: $31 - 4\left(3 - 9\right)$

Solution:

55

Exercise:

Problem: $24 - 3\left(2 - 10\right)$

Evaluate Variable Expressions with Integers

In the following exercises, evaluate each expression.

Exercise:

Problem: $x + 8$ when
ⓐ $x = -26$
ⓑ $x = -95$

Solution:

ⓐ -18 ⓑ -87

Exercise:

Problem: $y + 9$ when
ⓐ $y = -29$
ⓑ $y = -84$

Exercise:

Problem: When $b = -11$, evaluate:
ⓐ $b + 6$
ⓑ $-b + 6$

Solution:

ⓐ -5 ⓑ 17

Exercise:

Problem: When $c = -9$, evaluate:
ⓐ $c + \left(-4\right)$
ⓑ $-c + \left(-4\right)$

Exercise:

Problem: $p^2 - 5p + 2$ when
$p = -1$

Solution:

8

Exercise:

Problem: $q^2 - 2q + 9$ when $q = -2$

Exercise:

Problem: $6x - 5y + 15$ when $x = 3$ and $y = -1$

Solution:

38

Exercise:

Problem: $3p - 2q + 9$ when $p = 8$ and $q = -2$

Translate English Phrases to Algebraic Expressions

In the following exercises, translate to an algebraic expression and simplify if possible.

Exercise:

Problem: the sum of -4 and -17, increased by 32

Solution:

$(-4 + (-17)) + 32; 11$

Exercise:

Problem: ⓐ the difference of 15 and -7 ⓑ subtract 15 from -7

Exercise:

Problem: the quotient of -45 and -9

Solution:

$\frac{-45}{-9}; 5$

Exercise:

Problem: the product of -12 and the difference of c and d

Use Integers in Applications

In the following exercises, solve.

Exercise:

Problem:

Temperature The high temperature one day in Miami Beach, Florida, was 76°. That same day, the high temperature in Buffalo, New York was -8°. What was the difference between the temperature in Miami Beach and the temperature in Buffalo?

Solution:

84 degrees

Exercise:

Problem:

Checking Account Adrianne has a balance of $-\$22$ in her checking account. She deposits \$301 to the account. What is the new balance?

Visualize Fractions

Find Equivalent Fractions

In the following exercises, find three fractions equivalent to the given fraction. Show your work, using figures or algebra.

Exercise:

Problem: $\frac{1}{4}$

Solution:

$\frac{2}{8}$, $\frac{3}{12}$, $\frac{4}{16}$ answers may vary

Exercise:

Problem: $\frac{1}{3}$

Exercise:

Problem: $\frac{5}{6}$

Solution:

$\frac{10}{12}$, $\frac{15}{18}$, $\frac{20}{24}$ answers may vary

Exercise:

Problem: $\frac{2}{7}$

Simplify Fractions

In the following exercises, simplify.

Exercise:

Problem: $\frac{7}{21}$

Solution:

$\frac{1}{3}$

Exercise:

Problem: $\frac{8}{24}$

Exercise:

Problem: $\frac{15}{20}$

Solution:

$\frac{3}{4}$

Exercise:

Problem: $\frac{12}{18}$

Exercise:

Problem: $-\frac{168}{192}$

Solution:

$-\frac{7}{8}$

Exercise:

Problem: $-\frac{140}{224}$

Exercise:

Problem: $\frac{11x}{11y}$

Solution:

$\frac{x}{y}$

Exercise:

Problem: $\frac{15a}{15b}$

Multiply Fractions

In the following exercises, multiply.

Exercise:

Problem: $\frac{2}{5} \cdot \frac{1}{3}$

Solution:

$\frac{2}{15}$

Exercise:

Problem: $\frac{1}{2} \cdot \frac{3}{8}$

Exercise:

Problem: $\frac{7}{12}\left(-\frac{8}{21}\right)$

Solution:

$-\frac{2}{9}$

Exercise:

Problem: $\frac{5}{12}\left(-\frac{8}{15}\right)$

Exercise:

Problem: $-28p\left(-\frac{1}{4}\right)$

Solution:

$7p$

Exercise:

Problem: $-51q\left(-\frac{1}{3}\right)$

Exercise:

Problem: $\frac{14}{5}(-15)$

Solution:

-42

Exercise:

Problem: $-1\left(-\frac{3}{8}\right)$

Divide Fractions

In the following exercises, divide.

Exercise:

Problem: $\frac{1}{2} \div \frac{1}{4}$

Solution:

2

Exercise:

Problem: $\frac{1}{2} \div \frac{1}{8}$

Exercise:

Problem: $-\frac{4}{5} \div \frac{4}{7}$

Solution:

$-\frac{7}{5}$

Exercise:

Problem: $-\frac{3}{4} \div \frac{3}{5}$

Exercise:

Problem: $\frac{5}{8} \div \frac{a}{10}$

Solution:

$\frac{25}{4a}$

Exercise:

Problem: $\frac{5}{6} \div \frac{c}{15}$

Exercise:

Problem: $\frac{7p}{12} \div \frac{21p}{8}$

Solution:

$\frac{2}{9}$

Exercise:

Problem: $\frac{5q}{12} \div \frac{15q}{8}$

Exercise:

Problem: $\frac{2}{5} \div (-10)$

Solution:

$-\frac{1}{25}$

Exercise:

Problem: $-18 \div -\left(\frac{9}{2}\right)$

In the following exercises, simplify.

Exercise:

Problem: $\frac{\frac{2}{3}}{\frac{8}{9}}$

Solution:

$\frac{3}{4}$

Exercise:

Problem: $\frac{\frac{4}{5}}{\frac{8}{15}}$

Exercise:

Problem: $\frac{-\frac{9}{10}}{3}$

Solution:

$-\frac{3}{10}$

Exercise:

Problem: $\frac{2}{\frac{5}{8}}$

Exercise:

Problem: $\frac{\frac{r}{5}}{\frac{s}{3}}$

Solution:

$\frac{3r}{5s}$

Exercise:

Problem: $\frac{-\frac{x}{6}}{-\frac{8}{9}}$

Simplify Expressions Written with a Fraction Bar

In the following exercises, simplify.

Exercise:

Problem: $\frac{4+11}{8}$

Solution:

$\frac{15}{8}$

Exercise:

Problem: $\frac{9+3}{7}$

Exercise:

Problem: $\frac{30}{7-12}$

Solution:

-6

Exercise:

Problem: $\frac{15}{4-9}$

Exercise:

Problem: $\frac{22-14}{19-13}$

Solution:

$\frac{4}{3}$

Exercise:

Problem: $\frac{15+9}{18+12}$

Exercise:

Problem: $\frac{5\cdot 8}{-10}$

Solution:

-4

Exercise:

Problem: $\frac{3\cdot 4}{-24}$

Exercise:

Problem: $\frac{15\cdot 5-5^2}{2\cdot 10}$

Solution:

$\frac{5}{2}$

Exercise:

Problem: $\frac{12\cdot 9-3^2}{3\cdot 18}$

Exercise:

Problem: $\frac{2+4(3)}{-3-2^2}$

Solution:

-2

Exercise:

Problem: $\frac{7+3(5)}{-2-3^2}$

Translate Phrases to Expressions with Fractions

In the following exercises, translate each English phrase into an algebraic expression.

Exercise:

Problem: the quotient of *c* and the sum of *d* and 9.

Solution:

$\frac{c}{d+9}$

Exercise:

Problem: the quotient of the difference of *h* and *k*, and -5.

Add and Subtract Fractions

Add and Subtract Fractions with a Common Denominator

In the following exercises, add.
Exercise:

Problem: $\frac{4}{9}+\frac{1}{9}$

Solution:

$\frac{5}{9}$

Exercise:

Problem: $\frac{2}{9}+\frac{5}{9}$

Exercise:

Problem: $\frac{y}{3}+\frac{2}{3}$

Solution:

$\frac{y+2}{3}$

Exercise:

Problem: $\frac{7}{p}+\frac{9}{p}$

Exercise:

Problem: $-\frac{1}{8}+\left(-\frac{3}{8}\right)$

Solution:

$-\frac{1}{2}$

Exercise:

Problem: $-\frac{1}{8}+\left(-\frac{5}{8}\right)$

In the following exercises, subtract.
Exercise:

Problem: $\frac{4}{5}-\frac{1}{5}$

Solution:

$\frac{3}{5}$

Exercise:

Problem: $\frac{4}{5}-\frac{3}{5}$

Exercise:

Problem: $\frac{y}{17} - \frac{9}{17}$

Solution:

$\frac{y-9}{17}$

Exercise:

Problem: $\frac{x}{19} - \frac{8}{19}$

Exercise:

Problem: $-\frac{8}{d} - \frac{3}{d}$

Solution:

$-\frac{11}{d}$

Exercise:

Problem: $-\frac{7}{c} - \frac{7}{c}$

Add or Subtract Fractions with Different Denominators

In the following exercises, add or subtract.

Exercise:

Problem: $\frac{1}{3} + \frac{1}{5}$

Solution:

$\frac{8}{15}$

Exercise:

Problem: $\frac{1}{4} + \frac{1}{5}$

Exercise:

Problem: $\frac{1}{5} - \left(-\frac{1}{10}\right)$

Solution:

$\frac{3}{10}$

Exercise:

Problem: $\frac{1}{2} - \left(-\frac{1}{6}\right)$

Exercise:

Problem: $\frac{2}{3} + \frac{3}{4}$

Solution:

$\frac{17}{12}$

Exercise:

Problem: $\frac{3}{4}+\frac{2}{5}$

Exercise:

Problem: $\frac{11}{12}-\frac{3}{8}$

Solution:

$\frac{13}{24}$

Exercise:

Problem: $\frac{5}{8}-\frac{7}{12}$

Exercise:

Problem: $-\frac{9}{16}-\left(-\frac{4}{5}\right)$

Solution:

$\frac{19}{80}$

Exercise:

Problem: $-\frac{7}{20}-\left(-\frac{5}{8}\right)$

Exercise:

Problem: $1+\frac{5}{6}$

Solution:

$\frac{11}{6}$

Exercise:

Problem: $1-\frac{5}{9}$

Use the Order of Operations to Simplify Complex Fractions

In the following exercises, simplify.

Exercise:

Problem: $\frac{\left(\frac{1}{5}\right)^2}{2+3^2}$

Solution:

$\frac{1}{275}$

Exercise:

Problem: $\frac{\left(\frac{1}{3}\right)^2}{5+2^2}$

Exercise:

Problem: $\frac{\frac{2}{3}+\frac{1}{2}}{\frac{3}{4}-\frac{2}{3}}$

Solution:

14

Exercise:

Problem: $\frac{\frac{3}{4}+\frac{1}{2}}{\frac{5}{6}-\frac{2}{3}}$

Evaluate Variable Expressions with Fractions

In the following exercises, evaluate.

Exercise:

Problem: $x+\frac{1}{2}$ when
ⓐ $x=-\frac{1}{8}$
ⓑ $x=-\frac{1}{2}$

Solution:

ⓐ $\frac{3}{8}$ ⓑ 0

Exercise:

Problem: $x+\frac{2}{3}$ when
ⓐ $x=-\frac{1}{6}$
ⓑ $x=-\frac{5}{3}$

Exercise:

Problem: $4p^2q$ when $p=-\frac{1}{2}$ and $q=\frac{5}{9}$

Solution:

$\frac{5}{9}$

Exercise:

Problem: $5m^2n$ when $m=-\frac{2}{5}$ and $n=\frac{1}{3}$

Exercise:

Problem: $\frac{u+v}{w}$ when
$u=-4, v=-8, w=2$

Solution:

-6

Exercise:

Problem: $\frac{m+n}{p}$ when $m = -6, n = -2, p = 4$

Decimals

Name and Write Decimals

In the following exercises, write as a decimal.

Exercise:

Problem: Eight and three hundredths

Solution:

8.03

Exercise:

Problem: Nine and seven hundredths

Exercise:

Problem: One thousandth

Solution:

0.001

Exercise:

Problem: Nine thousandths

In the following exercises, name each decimal.

Exercise:

Problem: 7.8

Solution:

seven and eight tenths

Exercise:

Problem: 5.01

Exercise:

Problem: 0.005

Solution:

five thousandths

Exercise:

Problem: 0.381

Round Decimals

In the following exercises, round each number to the nearest ⓐ hundredth ⓑ tenth ⓒ whole number.

Exercise:

Problem: 5.7932

Solution:

ⓐ 5.79 ⓑ 5.8 ⓒ 6

Exercise:

Problem: 3.6284

Exercise:

Problem: 12.4768

Solution:

ⓐ 12.48 ⓑ 12.5 ⓒ 12

Exercise:

Problem: 25.8449

Add and Subtract Decimals

In the following exercises, add or subtract.

Exercise:

Problem: $18.37 + 9.36$

Solution:

27.73

Exercise:

Problem: $256.37 - 85.49$

Exercise:

Problem: $15.35 - 20.88$

Solution:

−5.53

Exercise:

Problem: $37.5 + 12.23$

Exercise:

Problem: $-4.2 + (-9.3)$

Solution:

−13.5

Exercise:

Problem: $-8.6 + (-8.6)$

Exercise:

Problem: $100 - 64.2$

Solution:

35.8

Exercise:

Problem: $100 - 65.83$

Exercise:

Problem: $2.51 + 40$

Solution:

42.51

Exercise:

Problem: $9.38 + 60$

Multiply and Divide Decimals

In the following exercises, multiply.

Exercise:

Problem: $(0.3)\,(0.4)$

Solution:

0.12

Exercise:

Problem: $(0.6)\,(0.7)$

Exercise:

Problem: $(8.52)\,(3.14)$

Solution:

26.7528

Exercise:

Problem: $(5.32)\,(4.86)$

Exercise:

Problem: $(0.09)\,(24.78)$

Solution:

2.2302

Exercise:

Problem: $(0.04)\,(36.89)$

In the following exercises, divide.

Exercise:

Problem: $0.15 \div 5$

Solution:

0.03

Exercise:

Problem: $0.27 \div 3$

Exercise:

Problem: $\$8.49 \div 12$

Solution:

$0.71

Exercise:

Problem: $\$16.99 \div 9$

Exercise:

Problem: $12 \div 0.08$

Solution:

150

Exercise:

Problem: $5 \div 0.04$

Convert Decimals, Fractions, and Percents

In the following exercises, write each decimal as a fraction.

Exercise:

Problem: 0.08

Solution:

$\frac{2}{25}$

Exercise:

Problem: 0.17

Exercise:

Problem: 0.425

Solution:

$\frac{17}{40}$

Exercise:

Problem: 0.184

Exercise:

Problem: 1.75

Solution:

$\frac{7}{4}$

Exercise:

Problem: 0.035

In the following exercises, convert each fraction to a decimal.

Exercise:

Problem: $\frac{2}{5}$

Solution:

0.4

Exercise:

Problem: $\frac{4}{5}$

Exercise:

Problem: $-\frac{3}{8}$

Solution:

-0.375

Exercise:

Problem: $-\frac{5}{8}$

Exercise:

Problem: $\frac{5}{9}$

Solution:

$0.\overline{5}$

Exercise:

Problem: $\frac{2}{9}$

Exercise:

Problem: $\frac{1}{2}+6.5$

Solution:

7

Exercise:

Problem: $\frac{1}{4}+10.75$

In the following exercises, convert each percent to a decimal.

Exercise:

Problem: 5%

Solution:

0.05

Exercise:

Problem: 9%

Exercise:

Problem: 40%

Solution:

0.4

Exercise:

Problem: 50%

Exercise:

Problem: 115%

Solution:

1.15

Exercise:

Problem: 125%

In the following exercises, convert each decimal to a percent.

Exercise:

Problem: 0.18

Solution:

18%

Exercise:

Problem: 0.15

Exercise:

Problem: 0.009

Solution:

0.9%

Exercise:

Problem: 0.008

Exercise:

Problem: 1.5

Solution:

150%

Exercise:

Problem: 2.2

The Real Numbers

Simplify Expressions with Square Roots

In the following exercises, simplify.

Exercise:

Problem: $\sqrt{64}$

Solution:

8

Exercise:

Problem: $\sqrt{144}$

Exercise:

Problem: $-\sqrt{25}$

Solution:

−5

Exercise:

Problem: $-\sqrt{81}$

Identify Integers, Rational Numbers, Irrational Numbers, and Real Numbers

In the following exercises, write as the ratio of two integers.

Exercise:

Problem: ⓐ 9 ⓑ 8.47

Solution:

ⓐ $\frac{9}{1}$ ⓑ $\frac{847}{100}$

Exercise:

Problem: ⓐ -15 ⓑ 3.591

In the following exercises, list the ⓐ rational numbers, ⓑ irrational numbers.

Exercise:

Problem: $0.84, 0.79132\ldots, 1.\overline{3}$

Solution:

ⓐ $0.84, 1.\overline{3}$ ⓑ $0.79132\ldots,$

Exercise:

Problem: $2.3\overline{8}, 0.572, 4.93814\ldots$

In the following exercises, identify whether each number is rational or irrational.

Exercise:

Problem: ⓐ $\sqrt{121}$ ⓑ $\sqrt{48}$

Solution:

ⓐ rational ⓑ irrational

Exercise:

Problem: ⓐ $\sqrt{56}$ ⓑ $\sqrt{16}$

In the following exercises, identify whether each number is a real number or not a real number.

Exercise:

Problem: ⓐ $\sqrt{-9}$ ⓑ $-\sqrt{169}$

Solution:

ⓐ not a real number ⓑ real number

Exercise:

Problem: ⓐ $\sqrt{-64}$ ⓑ $-\sqrt{81}$

In the following exercises, list the ⓐ whole numbers, ⓑ integers, ⓒ rational numbers, ⓓ irrational numbers, ⓔ real numbers for each set of numbers.

Exercise:

Problem: $-4, 0, \frac{5}{6}, \sqrt{16}, \sqrt{18}, 5.2537...$

Solution:

ⓐ $0, \sqrt{16}$ ⓑ $-4, 0, \sqrt{16}$ ⓒ $-4, 0, \frac{5}{6}, \sqrt{16}$ ⓓ $\sqrt{18}, 5.2537...$ ⓔ $-4, 0, \frac{5}{6}, \sqrt{16}, \sqrt{18}, 5.2537...$

Exercise:

Problem: $-\sqrt{4}, 0.\overline{36}, \frac{13}{3}, 6.9152..., \sqrt{48}, 10\frac{1}{2}$

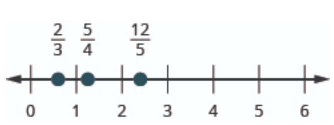
Locate Fractions on the Number Line

In the following exercises, locate the numbers on a number line.

Exercise:

Problem: $\frac{2}{3}, \frac{5}{4}, \frac{12}{5}$

Solution:



Exercise:

Problem: $\frac{1}{3}, \frac{7}{4}, \frac{13}{5}$

Exercise:

Problem: $2\frac{1}{3}, -2\frac{1}{3}$

Solution:

Exercise:

Problem: $1\frac{3}{5}, -1\frac{3}{5}$

In the following exercises, order each of the following pairs of numbers, using $<$ or $>$.

Exercise:

Problem: -1 ___ $-\frac{1}{8}$

Solution:

$<$

Exercise:

Problem: $-3\frac{1}{4}$ ___ -4

Exercise:

Problem: $-\frac{7}{9}$ ___ $-\frac{4}{9}$

Solution:

$>$

Exercise:

Problem: -2 ___ $-\frac{19}{8}$

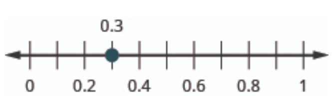
Locate Decimals on the Number Line

In the following exercises, locate on the number line.

Exercise:

Problem: 0.3

Solution:



Exercise:

Problem: -0.2

Exercise:

Problem: -2.5

Solution:

Exercise:

Problem: 2.7

In the following exercises, order each of the following pairs of numbers, using < or >.

Exercise:

Problem: $0.9\_\_\_0.6$

Solution:

$>$

Exercise:

Problem: $0.7\_\_\_0.8$

Exercise:

Problem: $-0.6\_\_\_-0.59$

Solution:

$>$

Exercise:

Problem: $-0.27\_\_\_-0.3$

Properties of Real Numbers

Use the Commutative and Associative Properties

In the following exercises, use the Associative Property to simplify.

Exercise:

Problem: $-12\left(4m\right)$

Solution:

$-48m$

Exercise:

Problem: $30\left(\frac{5}{6}q\right)$

Exercise:

Problem: $(a+16)+31$

Solution:

$a+47$

Exercise:

Problem: $(c+0.2)+0.7$

In the following exercises, simplify.

Exercise:

Problem: $6y+37+(-6y)$

Solution:

37

Exercise:

Problem: $\frac{1}{4}+\frac{11}{15}+\left(-\frac{1}{4}\right)$

Exercise:

Problem: $\frac{14}{11}\cdot\frac{35}{9}\cdot\frac{14}{11}$

Solution:

$\frac{35}{9}$

Exercise:

Problem: $-18\cdot 15\cdot\frac{2}{9}$

Exercise:

Problem: $\left(\frac{7}{12}+\frac{4}{5}\right)+\frac{1}{5}$

Solution:

$1\frac{7}{12}$

Exercise:

Problem: $(3.98d+0.75d)+1.25d$

Exercise:

Problem: $11x+8y+16x+15y$

Solution:

$27x+23y$

Exercise:

Problem: $52m+(-20n)+(-18m)+(-5n)$

Use the Identity and Inverse Properties of Addition and Multiplication

In the following exercises, find the additive inverse of each number.

Exercise:

Problem:
ⓐ $\frac{1}{3}$
ⓑ 5.1
ⓒ -14
ⓓ $-\frac{8}{5}$

Solution:

ⓐ $-\frac{1}{3}$ ⓑ -5.1 ⓒ 14 ⓓ $\frac{8}{5}$

Exercise:

Problem:
ⓐ $-\frac{7}{8}$
ⓑ -0.03
ⓒ 17
ⓓ $\frac{12}{5}$

In the following exercises, find the multiplicative inverse of each number.

Exercise:

Problem: ⓐ 10 ⓑ $-\frac{4}{9}$ ⓒ 0.6

Solution:

ⓐ $\frac{1}{10}$ ⓑ $-\frac{9}{4}$ ⓒ $\frac{5}{3}$

Exercise:

Problem: ⓐ $-\frac{9}{2}$ ⓑ -7 ⓒ 2.1

Use the Properties of Zero

In the following exercises, simplify.

Exercise:

Problem: $83\cdot 0$

Solution:

0

Exercise:

Problem: $\frac{0}{9}$

Exercise:

Problem: $\frac{5}{0}$

Solution:

undefined

Exercise:

Problem: $0 \div \frac{2}{3}$

In the following exercises, simplify.

Exercise:

Problem: $43 + 39 + (-43)$

Solution:

39

Exercise:

Problem: $(n + 6.75) + 0.25$

Exercise:

Problem: $\frac{5}{13} \cdot 57 \cdot \frac{13}{5}$

Solution:

57

Exercise:

Problem: $\frac{1}{6} \cdot 17 \cdot 12$

Exercise:

Problem: $\frac{2}{3} \cdot 28 \cdot \frac{3}{7}$

Solution:

8

Exercise:

Problem: $9\,(6x - 11) + 15$

Simplify Expressions Using the Distributive Property

In the following exercises, simplify using the Distributive Property.

Exercise:

Problem: $7\left(x+9\right)$

Solution:

$7x+63$

Exercise:

Problem: $9\left(u-4\right)$

Exercise:

Problem: $-3\left(6m-1\right)$

Solution:

$-18m+3$

Exercise:

Problem: $-8\left(-7a-12\right)$

Exercise:

Problem: $\frac{1}{3}\left(15n-6\right)$

Solution:

$5n-2$

Exercise:

Problem: $\left(y+10\right)\cdot p$

Exercise:

Problem: $\left(a-4\right)-\left(6a+9\right)$

Solution:

$-5a-13$

Exercise:

Problem: $4\left(x+3\right)-8\left(x-7\right)$

Systems of Measurement

1.1 Define U.S. Units of Measurement and Convert from One Unit to Another

In the following exercises, convert the units. Round to the nearest tenth.

Exercise:

Problem: A floral arbor is 7 feet tall. Convert the height to inches.

Solution:

84 inches

Exercise:

Problem: A picture frame is 42 inches wide. Convert the width to feet.

Exercise:

Problem: Kelly is 5 feet 4 inches tall. Convert her height to inches.

Solution:

64 inches

Exercise:

Problem: A playground is 45 feet wide. Convert the width to yards.

Exercise:

Problem: The height of Mount Shasta is 14,179 feet. Convert the height to miles.

Solution:

2.7 miles

Exercise:

Problem: Shamu weights 4.5 tons. Convert the weight to pounds.

Exercise:

The play lasted

$$1\frac{3}{4}$$

Problem: hours. Convert the time to minutes.

Solution:

105 minutes

Exercise:

Problem: How many tablespoons are in a quart?

Exercise:

Problem: Naomi’s baby weighed 5 pounds 14 ounces at birth. Convert the weight to ounces.

Solution:

94 ounces

Exercise:

Problem: Trinh needs 30 cups of paint for her class art project. Convert the volume to gallons.

Use Mixed Units of Measurement in the U.S. System.

In the following exercises, solve.

Exercise:

Problem:

John caught 4 lobsters. The weights of the lobsters were 1 pound 9 ounces, 1 pound 12 ounces, 4 pounds 2 ounces, and 2 pounds 15 ounces. What was the total weight of the lobsters?

Solution:

10 lbs. 6 oz.

Exercise:

Problem:

Every day last week Pedro recorded the number of minutes he spent reading. The number of minutes were 50, 25, 83, 45, 32, 60, 135. How many hours did Pedro spend reading?

Exercise:

Problem:

Fouad is 6 feet 2 inches tall. If he stands on a rung of a ladder 8 feet 10 inches high, how high off the ground is the top of Fouad’s head?

Solution:

15 feet

Exercise:

Problem:

Dalila wants to make throw pillow covers. Each cover takes 30 inches of fabric. How many yards of fabric does she need for 4 covers?

Make Unit Conversions in the Metric System

In the following exercises, convert the units.

Exercise:

Problem: Donna is 1.7 meters tall. Convert her height to centimeters.

Solution:

170 centimeters

Exercise:

Problem: Mount Everest is 8,850 meters tall. Convert the height to kilometers.

Exercise:

Problem: One cup of yogurt contains 488 milligrams of calcium. Convert this to grams.

Solution:

0.488 grams

Exercise:

Problem: One cup of yogurt contains 13 grams of protein. Convert this to milligrams.

Exercise:

Problem: Sergio weighed 2.9 kilograms at birth. Convert this to grams.

Solution:

2,900 grams

Exercise:

Problem: A bottle of water contained 650 milliliters. Convert this to liters.

Use Mixed Units of Measurement in the Metric System

In the following exerices, solve.

Exercise:

Problem:

Minh is 2 meters tall. His daughter is 88 centimeters tall. How much taller is Minh than his daughter?

Solution:

1.12 meter

Exercise:

Problem:

Selma had a 1 liter bottle of water. If she drank 145 milliliters, how much water was left in the bottle?

Exercise:

Problem:

One serving of cranberry juice contains 30 grams of sugar. How many kilograms of sugar are in 30 servings of cranberry juice?

Solution:

0.9 kilograms

Exercise:

Problem:

One ounce of tofu provided 2 grams of protein. How many milligrams of protein are provided by 5 ounces of tofu?

Convert between the U.S. and the Metric Systems of Measurement

In the following exercises, make the unit conversions. Round to the nearest tenth.
Exercise:

Problem: Majid is 69 inches tall. Convert his height to centimeters.

Solution:

175.3 centimeters

Exercise:

Problem: A college basketball court is 84 feet long. Convert this length to meters.

Exercise:

Problem: Caroline walked 2.5 kilometers. Convert this length to miles.

Solution:

1.6 miles

Exercise:

Problem: Lucas weighs 78 kilograms. Convert his weight to pounds.

Exercise:

Problem: Steve's car holds 55 liters of gas. Convert this to gallons.

Solution:

14.6 gallons

Exercise:

Problem: A box of books weighs 25 pounds. Convert the weight to kilograms.

Convert between Fahrenheit and Celsius Temperatures

In the following exercises, convert the Fahrenheit temperatures to degrees Celsius. Round to the nearest tenth.
Exercise:

Problem: 95° Fahrenheit

Solution:

35° C

Exercise:

Problem: 23° Fahrenheit

Exercise:

Problem: 20° Fahrenheit

Solution:

–6.7° C

Exercise:

Problem: 64° Fahrenheit

In the following exercises, convert the Celsius temperatures to degrees Fahrenheit. Round to the nearest tenth.

Exercise:

Problem: 30° Celsius

Solution:

86° F

Exercise:

Problem: –5° Celsius

Exercise:

Problem: –12° Celsius

Solution:

10.4° F

Exercise:

Problem: 24° Celsius

Chapter Practice Test

Exercise:

Problem: Write as a whole number using digits: two hundred five thousand, six hundred seventeen.

Solution:

205,617

Exercise:

Problem: Find the prime factorization of 504.

Exercise:

Problem: Find the Least Common Multiple of 18 and 24.

Solution:

72

Exercise:

Problem: Combine like terms: $5n + 8 + 2n - 1$.

In the following exercises, evaluate.
Exercise:

Problem: $-|x|$ when $x=-2$

Solution:

-2

Exercise:

Problem: $11-a$ when $a=-3$

Exercise:

Problem: Translate to an algebraic expression and simplify: twenty less than negative 7.

Solution:

$-7-20;-27$

Exercise:

Problem:

Monique has a balance of $-\$18$ in her checking account. She deposits $152 to the account. What is the new balance?

Exercise:

Problem: Round 677.1348 to the nearest hundredth.

Solution:

677.13

Exercise:

Problem: Convert $\frac{4}{5}$ to a decimal.

Exercise:

Problem: Convert 1.85 to a percent.

Solution:

185%

Exercise:

Problem: Locate $\frac{2}{3}$, -1.5, and $\frac{9}{4}$ on a number line.

In the following exercises, simplify each expression.
Exercise:

Problem: $4+10\left(3+9\right)-5^2$

Solution:

99

Exercise:

Problem: $-85 + 42$

Exercise:

Problem: $-19 - 25$

Solution:

-44

Exercise:

Problem: $(-2)^4$

Exercise:

Problem: $-5\,(-9) \div 15$

Solution:

3

Exercise:

Problem: $\frac{3}{8} \cdot \frac{11}{12}$

Exercise:

Problem: $\frac{4}{5} \div \frac{9}{20}$

Solution:

$\frac{16}{9}$

Exercise:

Problem: $\frac{12+3\cdot5}{15-6}$

Exercise:

Problem: $\frac{m}{7} + \frac{10}{7}$

Solution:

$\frac{m+10}{7}$

Exercise:

Problem: $\frac{7}{12} - \frac{3}{8}$

Exercise:

Problem: $-5.8 + (-4.7)$

Solution:

-10.5

Exercise:

Problem: $100 - 64.25$

Exercise:

Problem: $(0.07)(31.95)$

Solution:

2.2365

Exercise:

Problem: $9 \div 0.05$

Exercise:

Problem: $-14\left(\frac{5}{7}p\right)$

Solution:

$-10p$

Exercise:

Problem: $(u+8)-9$

Exercise:

Problem: $6x + (-4y) + 9x + 8y$

Solution:

$15x + 4y$

Exercise:

Problem: $\frac{0}{23}$

Exercise:

Problem: $\frac{75}{0}$

Solution:

undefined

Exercise:

Problem: $-2(13q - 5)$

Exercise:

Problem: A movie lasted $1\frac{2}{3}$ hours. How many minutes did it last? *(1 hour = 60 minutes)*

Solution:

100 minutes

Exercise:

Problem:

Mike’s SUV is 5 feet 11 inches tall. He wants to put a rooftop cargo bag on the the SUV. The cargo bag is 1 foot 6 inches tall. What will the total height be of the SUV with the cargo bag on the roof? *(1 foot = 12 inches)*

Exercise:

Problem: Jennifer ran 2.8 miles. Convert this length to kilometers. *(1 mile = 1.61 kilometers)*

Solution:

4.508 km